Invariant Theorems in the Euclidean Geometry with Respect to Conics

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A research is usually preceded by a natural aspiration of people to find new knowledge on the base of well-known facts. A large number of scientific facts are known for centuries, however new details and peculiarities are discovered in connection with them due to contemporary technologies. Information technologies based on specialized software propose rich possibilities for the development of new thinking. The software itself together with abundant arsenal of knowledge and skills for their effective application initiates specific way of thinking. Thus, the dynamic geometric software turns out to be a basic instrument to study the objects that belong to the classic Euclidean geometry. Various software products exist to perform corresponding investigations in the domain under consideration. The present talk uses the possibilities of the dynamic geometric software "THE GEOMETERS SKETCHPAD" (GSP) in the generalization of some classic but also of some not very popular theorems from the triangle geometry. The generalizations are realized in analyzing basic properties of the geometric objects under study.

The challenge to find a generalization of a geometric theorem is connected with a deep understanding of the considered figure properties. A necessary step is to clarify the relation among the elements of a given configuration, thus extracting the properties which could be changed. How to perform the change? Which elements and properties should be modified in order to change the corresponding theorem itself? The GSP program turns out to be a useful instrument in the process of answering these questions. The theorems included in the talk are mostly from the triangle geometry and are connected with different classes of circles, lines and points in the plane of that triangle. After the analysis of the corresponding relations the circles, the lines and the points are replaced by suitable conics, lines and points, thus keeping the validity of the theorems in the new situation. A deep knowledge of conic properties and constructive skills are necessary for the purpose. The program GSP is applied for fast elimination of various conjectures which turn out to be false, but also for the creation of convincing configurations leading to the formulation of the desired assertions. The assertions themselves should be considered as true only in case they are strictly proven. Generalizations are obtained in many cases but reasons are found very often to reject some.

The established generalizations propose a new view on well-known geometric theorems and expose a deeper sense of the participating figures. They give possibilities to overcome the limits of previous perceptions. Thus, a gradual deepening of the understanding concerning the projective properties of conics is realized. Experience is obtained to discover certain theorems, which helps further investigations making them easier.