Some non-standard pseudo-calculus in the treatment of dispersive equations

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Micro-local analysis, in particular, the theory of pseudo-differential or Fourier integral operators is an important tool in the treatment of qualitative properties of solutions to partial differential equations. We consider two Cauchy problems for dispersive equations. First, we are interested in the Cauchy problem

$$u_{tt} - \sum_{k,l=1}^{n} a_{k,l}(t,x)u_{x_k x_l} = f(t,x), \;\; u(0,x) = \varphi(x), \;\; u_t(0,x) = \psi(x).$$

Here the coefficients are smooth in the spatial variables, but not necessarily Lipschitz continuous in the time variable. Our goal is the construction of a parametrix. The second model is the Cauchy problem for the degenerate Schrödinger equation

$$iu_t + t^l \Delta_x u + \sum_{j=1}^{n} t^k b_j(t,x) u_{x_j} = 0, \;\; u(0,x) = \varphi(x).$$

Here our goal is to study well-posedness in Gevrey spaces, in particular, the influence of Levi conditions on the Gevrey exponent. For both problems we develop a non-standard pseudo-calculus basing on a division of the extended phase space into zones.