The edge Folkman number $F_e(3,3;4)$ is greater than 19

ALEKSANDAR BIKOV, NEDYALKO NENOV
Faculty of Mathematics and Informatics,
Sofia University "St. Kliment Ohridski", Sofia, Bulgaria
asbikov@fmi.uni-sofia.bg, nenov@fmi.uni-sofia.bg

We denote by $\mathcal{H}_e(3,3;q)$ the set of graphs which have the property that in every coloring of their edges in two colors there is a monochromatic triangle and they do not contain a complete subgraph on q vertices K_q. The minimum number of vertices of graphs in $\mathcal{H}_e(3,3;q)$ is denoted by $F_e(3,3;q)$ and it is called an edge Folkman number. Folkman proved in [1] that $F_e(3,3;q)$ exists if and only if $q \geq 4$. It is well known that $F_e(3,3;q) = 6$ if $q \geq 7$. We also know that $F_e(3,3;6) = 8$ [2] and that $F_e(3,3;5) = 15$. The upper bound $F_e(3,3;5) \leq 15$ is proved in [4] and the lower bound is obtained in [5] with the help of computer. The exact value of the number $F_e(3,3;4)$ is not known. For now we know that this number is between 19 and 786, [6] and [3]. By improving the algorithms used in [6] we obtain the new bound $F_e(3,3;4) \geq 20$.

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