BOOK OF ABSTRACTS

V Congress of Mathematicians of Macedonia

Ohrid, R. Macedonia, September 24-27, 2014

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V Congress of Mathematicians of Macedonia
Book of Abstracts

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This book contains all the abstracts of the talks accepted for the Congress. The abstracts of the plenary speakers and of the sectional plenary speakers are given in alphabetical order. The abstracts of the short communications are listed by sections, in alphabetical order of the presenters of the talks.

Edited by:
Vesna Celakoska-Jordanova

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Welcome Message

I am honored and delighted to welcome you to the largest international congress in the mathematics community in Republic of Macedonia organized by the Union of Mathematicians of Macedonia. Since 1996, there have been 4 congresses of mathematicians of Macedonia: the First (1996) and the Second Congress (2000) were held in Ohrid, the Third (2005) and the Fourth Congress (2008) were held in Struga.

The purpose of the Fifth Congress is to bring together the members of the mathematical community, researches and educators of mathematics from Macedonia and all over the world, and to enable, in a 4-day comprehensive program that includes invited lectures, contributed talks and workshops, the exchange of ideas and knowledge. I believe that we have chosen a venue that guaranties a successful congress meeting and hope that you will also be able to savor some of the fine attractions that our country offers. A visit to Ohrid numerous historical relics, some of which are designated UNESCO World Cultural Heritage Sites, will make the congress participants journey all the more special.

Welcome at the V Congress 2014!

The President of the
Union of Mathematicians of Macedonia

Aleksa Malčeski

I Plenary Speakers

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Arithmetics and Harmonic Analysis

Vladimir N. Chubarikov

Trigonometric integrals and trigonometric sums. The singular integrals and sums of additive problems of the number theory. Distributions of values of arithmetical functions.

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Fragmentability of function spaces $C_p(T)$ for pseudocompact spaces T

Petar S. Kenderov

A metric $d(.,.)$ defined in a topological space X is said to *fragment* X, if for every $\varepsilon > 0$ and every non-empty subset $A \subset X$ there exists an open subset $U \subset X$ such that the set $A \cap U$ is not empty and its d-diameter is smaller than ε. I.e. every non-empty set $A \subset X$ contains relatively open subsets of arbitrarily small diameters. The space X is said to be *fragmentable* if there exists a metric that fragments it. Fragmentability was introduced by Jayne and Rogers (see [1]) and studied by many authors. It proved to be a convenient tool in the study of Banach spaces, differentiability of convex functions as well as in many topological contexts (see Jayne, Namioka and Rogers [2]-[6], Ribarska [11]-[13], Namioka [10], [11] and Kenderov, Moors [7], [8]). Of particular interest is the case when the open subsets of X are open in the metric topology generated by the metric d. In such a case it is said that *d majorizes the topology of X*.

For function spaces $C(T)$, where T is compact, it has been shown (see [8] and [7]) that the pointwise convergence topology p is fragmented by a majorizing metric d if, and only if, there exists another fragmenting metric $d\,'$ which majorizes even the uniform convergence topology (the one generated by the "sup-norm" in $C(T)$). The major goal of this study, which is a joint work with M. M. Choban and W. B. Moors, is to show that this result remains valid for pseudocompact spaces T as well.

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Special Session

Multiplication of distributions: history and applications

Michael Oberguggenberger

Motivated by problems in quantum field theory, the first proposals for defining a product of distributions were made in the 1950ies. While up to the 1970ies most efforts were directed towards defining individual products, the theory matured in the 1980ies with the development of differential algebras containing the space of distributions - based on sequences of regularization with various asymptotic properties.

By now, algebras of generalized functions have been successfully applied to nonlinear partial differential equations with distribution data, to linear partial differential equations with non-smooth coefficients, to stochastic analysis, to semi-Riemannian geometry with non-smooth metrics, to regularity theory, among other areas. Further, concrete applications to physics and other sciences have been established as well.

It is the purpose of this talk to isolate the historical lines of development and to highlight some of the recent applications in the mentioned fields.

Mathematics Subject Classification 2010: 46F30.

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Shadowing and the global theory of dynamical systems

Sergei Yu. Pilyugin

In this talk, we discuss the following topics:

- shadowing and structural stability;
- genericity of various shadowing properties;
- shadowing in systems without hyperbolicity.

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Wave fronts opposite to the quasi-analytic properties of distributions

Stevan Pilipović

Quasi-analytic wave-front sets of distributions which correspond to the Gevrey sequence $p!^s$, $s \in [1/2, 1)$ are defined and investigated. The localization by a cut-off function, for distributions and ultra-distributions which correspond to $p!^s$, $s > 1$, or the localization by a sequence of cut-off functions in the analytic case ($s = 1$) and then the use of Fourier transform is replaced, in the case $s \in [1/2, 1)$, by a new approach based on a restriction of a distribution, then extension of this restriction and the use of a sequence of short time Fourier transforms with the Gaussian window functions. Then, the basic properties are established.

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Topological methods in dynamics

José M.R. Sanjurjo

We shall consider some situations where methods of algebraic topology and shape theory are useful in dynamical systems, in particular in the Conley index theory, in the study of attractors and in population dynamics.

As an example, we shall study the internal structure of the global attractor of a uniformly persistent flow and show that the restriction of the flow to the global attractor has duality properties which can be expressed in terms of cer- tain attractor-repeller decompositions. We shall also study some natural Morse decompositions of the flow and calculate their Morse equations. These equations provide necessary and sufficient conditions for the existence of attractors with spherical shape or such that their suspension has spherical shape.

We shall study generalized Poincaré-Andronov-Hopf bifurcations of parameterized families of flows at boundary points of an n-dimensional manifold and see that this kind of bifurcations produce a whole family of attractors evolving from the bifurcation point and having interesting topological properties. We shall show that, in some cases, the bifurcation transforms a system with extreme non-permanence properties into a uniformly persistent one. We study in this talk the circumstances in which this phenomenon happens and provide an example constructed by combining a Holling-type interaction with a pitchfork bifurcation.

Mathematics Subject Classification 2010: 37B30, 55P55.

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Heuristic tools for the generation of new mathematical facts

Sava Grozdev

Some geometric configurations are considered in connection with circles and related generalizations are proposed including second-order curves, mainly conics. Several approaches are discussed to compose new mathematical facts. One of them is to generalize one or some of the initial conditions. For example, if a problem is connected with a right angled triangle, a natural idea is to examine what will happen if the triangle is arbitrary. Computer animations are used for the purpose. The new situation may need changes in the initial configuration. Animations help to examine them. Another approach refers to situations which allow consideration of objects or groups of objects which exist in a given fact but are not connected directly with it. Computer animations are also productive in such an approach. Examples are presented, referring to the last several years IMO Geometry problems, which are of high quality and content, proposing possibilities for further investigations and generalizations. In a corresponding research process accompanying results appear in a natural way. The software program "THE GEOMETER'S SKETCHPAD" (GSP) is applied as a heuristic tool for the purpose.

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II Sectional Plenary Speakers

Dragan S. Djordjević

Operators on Hilbert C^*-modules

Yury T. Lisica

On all numbers great and small

Zoran Rakić

On Osserman condition and duality principle

Jasson Vindas

A general integral

Operators on Hilbert C^*-modules

Dragan S. Djordjević

We present results related to bounded andjointable operators on Hilbert C^*-modules. Results concerning generalized inverses are included.

Mathematics Subject Classification 2010: 46L08.

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On all numbers great and small

Yury T. Lisica

An original construction of numbers great and small was given by Conway in [1] with the purpose to connect it with games. Two ideas were used for it, i.e., Mirimanoff's representation of the ordinal numbers: $0 = \{\}$, $1 = \{0\}$, $2 = \{0, \{0\}\}$, $3 = \{0, 1, \{0, 1\}\}$ and so on, and Dedekind sections in $\mathbb{Q}$ for the construction of the real numbers $\mathbb{R}$.

Conway's definition of number is the following: if L and R are any two *sets* of numbers, and *no member of* L *is* $\geq$ *any member of* R, then there is a number $\{L|R\}$. If $x = \{L|R\}$ is a number, then for short $x = \{x^L|x^R\}$, where x^L is a typical member of L and x^R is a typical member of R, i.e. $L = \{x^L\}$ and $R = \{x^R\}$.

Further $x \leq y$ iff no $x^R \leq y$ and $x \leq$ no y^L as well as $x = y$ iff $x \geq y$ and $y \geq x$. Clearly, what $x > y$ is and one can see that $x > x^L$ and $x < x^R$ for each $x = \{x^L|x^R\}$ and all its components x^L and x^R. Note that different representations $\{L'|R'\}$ and $\{L|R\}$ can define the same number; that is why one must distinguish between the form $\{L|R\}$ of a number and the number itself.

By definition, $x + y = \{x^L + y, x + y^L|x^R + y, x + y^R\}$, $-x = \{-x^R|-x^L\}$ and $xy = \{x^L y + xy^L - x^L y^L, x^R y + xy^R - x^R y^R|x^L y + xy^R - x^L y^R, x^R y + xy^L - x^R y^L\}$.

So, the proper Class No of all numbers, which turns out to be a universally embedding totally ordered Field (i.e., a field whose domain is a proper Class) containing the real numbers (with usual operations $x + y$ and xy) and the ordinal numbers (with maximal or natural sum and product, not the usual ordinal operations), is constructed by the following induction:
$0 = \{|\} = \{-1|\} = \{|1\} = \{-1|1\}$ (born on day 0),
$1 = \{0|\} = \{-1, 0|\}$ and $-1 = \{|0\} = \{|0, 1\}$ (born on day 1),
$2 = \{0, 1|\}$, $1/2 = \{0|1\}$, $-1/2 = \{-1|0\}$, $-2 = \{|-1, 0, \}$ (born on day 2)
...
$\omega = \{0, 1, 2, 3, ...|\}$, π, e, $\sqrt{2}$, $1/3$, $1/\omega = \{0|1, 1/2, 1/4, 1/8, ...\}$,
$-\omega = \{|0, -1, -2, -3, ...\}$ (born on day ω),
$\omega + 1 = \{0, 1, 2, 3, ..., \omega|\}$, $\omega - 1 = \{0, 1, 2, 3, ..., |\omega\}$, $\sqrt{2} + 1/\omega$, $\sqrt{2} - 1/\omega$,
$2/\omega = \{1/\omega|1, 1/2, 1/4, ...\}$, $1/2\omega = \{0, |1/\omega\}$ (born on day $\omega + 1$),
...
$\omega 2 = \{0, 1, 2, 3, ...\omega, \omega + 1, \omega + 2, ...|\}$, $\omega/2 = \{0, 1, 2, 3, ...|\omega, \omega - 1, \omega - 2, ...\}$,
$\sqrt{\omega} = \{0, 1, 2, 3, ...|\omega, \omega/2, \omega/4, ...\}$,
$1/\omega^2 = \{0|1/\omega, 1/2\omega, 1/4\omega, ...\}$ (born on day $\omega 2$)
and so on for all ordinal numbers α.

The purpose of this talk is to consider different subsets and subfields of No together with linear ordering topologies on them and to study two possible completions of them which are actually subsets of No whether one of them is a subfield and another only a "manifold". In [1] subfields of No except $\mathbb{R}$ were not considered. In addition, we take the Dedekind sections $\{L|R\}$ in No where L and R are not sets but proper classes and thus $\{L|R\}$ are not numbers. Nevertheless, one can call some of them transdefinite numbers and all these transdefinite numbers together with No form a completion of No with transdefinite field structure. Together with gaps that are not numbers at all these Dedekind sections and No form a transdefinite continuous one-dimensional straight line which is a completion of

No. Using the method in [2], one can define the convergence of a positive series $\sum\limits_{0\leq\alpha<\lambda} x_\alpha$, where $\lambda > \omega$ and $x_\alpha \in No$.

Let ω_ξ be an initial ordinal, where ξ is an ordinal number ≥ 0. Let $W(\omega_\xi)$ be the set of all ordinals $\alpha < \omega_\xi$ and P_ξ be a localization in zero of the ring of all Conway's numbers born before day ω_ξ. By Ω we denote the class of all ordinals, which was called in [2] *eschaton*; $\Omega = \{0, 1, 2, ..., \alpha, ...|\}$. Denote by N_ξ the set of all ordinals $\alpha < \omega_\xi$ with operations $\alpha + \beta$ and $\alpha\beta$ as the natural sum and product. We extend N_ξ to $Z_\xi = N_\xi \cup (-N_\xi)$ which turns out to be a commutative ring without zero divisor, and let Q_ξ be a localization in zero of Z_ξ, actually. P_ξ and Q_ξ satisfies the following generalized Archimedean property: $\forall x, y > 0$ in P_ξ (in Q_ξ, resp.) $\exists$ an ordinal number $\alpha \in N_\xi$ such that $x\alpha > y$.

A transfinite sequence (x_α), $0 \leq \alpha < \omega_\xi$, i.e., a mapping $\varphi : W(\omega_\xi) \to P_\xi$, is called a fundamental sequence in P_ξ if for each positive number $\varepsilon \in P_\xi$ there is an ordinal number α_0 such that $|x_\alpha - x_{\alpha'}| < \varepsilon$ for all $\alpha, \alpha' > \alpha_0$ ($|x_\alpha - x_{\alpha'}|$ is the absolute value of $x_\alpha - x_{\alpha'}$). Two fundamental sequences $(x_\alpha), (y_\alpha)$ are equivalent if for each positive number $\varepsilon \in P_\xi$ there is an ordinal number α_0 such that $|x_\alpha - y_\alpha| < \varepsilon$ for all $\alpha > \alpha_0$. A sequence (x_α) is called convergent if it is equivalent to the constant sequence (a); if $(a = 0)$, then (x_α) is called infinitesimal. Each convergent sequence is fundamental; converse is not true in general. But each fundamental sequence (x_α) defines a Conway's number $x = \{L|R\}$, where $L = \{l \in P_\xi | (\exists\alpha_0)(\forall\alpha > \alpha_0)(l < x_\alpha)\}$ and $R = \{r \in P_\xi | (\exists\alpha_0)(\forall\alpha > \alpha_0)(x_\alpha < r)\}$. We call a fundamental sequence (x_α) in P_ξ two-sided if there exist sequences (x^L_α) in L and (x^R_α) in R equivalent to (x_α).

One can consider the similar definitions for sequences (x_α) in Q_ξ.

Suppose now that the power $\aleph_\xi$ of ω_ξ is a regular cardinal number. Denote by $\mathbb{R}_\xi$ and R_ξ the classes $[(x_\alpha)]$ of all equivalent fundamental two-sided sequences in Q_ξ and P_ξ, respectively, and define the operations $+$ and $\cdot$ via sum and product of two-sided sequences in corresponding classes.

Theorem 1. *$\mathbb{R}_\xi$ is a subfield of No and $dim\mathbb{R}_\xi = 0$, $\xi > 0$, $dim\mathbb{R}_0 = 1$. R_ξ is a subfield of No and $dimR_\xi = 0$,$\xi > 0$, $dimR_0 = 1$. Moreover, $\mathbb{R}_\xi \subseteq R_\xi$.*

Theorem 2. *If L_ξ is the set of all Dedekind sections in P_ξ, then $dimL_\xi = 1$ in the corresponding linear ordering topologies, $\xi \geq 0$. Each bounded set $X \subset L_\xi$ has a smallest upper bound $\sup X$ and a greatest low bound $\inf X$ in L_ξ.*

Theorem 3. *The corresponding transdefinite completions R_Ω and L_Ω of No have the same properties, i.e. $dimR_\Omega = 0$ and $dimL_\Omega = 1$. Moreover, R_Ω is a transdefinite Field and L_Ω is a transdefinite "Manifold".*

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On Osserman condition and duality principle

Zoran Rakić

Let (M, g) be a pseudo-Riemannian manifold, with curvature tensor R. The Jacobi operator R_X is the symmetric endomorphism of T_pM defined by $R_X(Y) = R(Y, X)X$. In Riemannian settings, if M is locally a rank-one symmetric space or if M is flat, then the eigenvalues of R_X are constant on SM. Osserman wondered if the converse held; this question is usually known as the *Osserman conjecture*.

In the last twenty years many authors have been studied problems which arising from the Osserman conjecture and its generalizations. In the first part of the lecture we will give an overview of Osserman type problems in the psuedo-Riemannian geometry. The second part is devoted to the equivalence of the Osserman pointwise condition and the duality principle. This part of the lecture consists new results, which are obtained in collaboration with Yury Nikolayevsky and Vladica Andrejić.

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A general integral

Jasson Vindas

In this talk we present the construction of a new integral, the distributional integral of functions of one real variable, that is more general than the Lebesgue and the Denjoy-Perron-Henstock-Kurzweil integrals, and which also allows the integration of functions with distributional values everywhere or nearly everywhere.

The distributional integral is more general than the standard integrals, but it still has many of the useful properties of those standard ones, including integration by parts formulas, substitution formulas, even for infinite intervals –in the Cesàro sense–, mean value theorems, and convergence theorems. The distributional integral satisfies a version of Hake's theorem.

Our integral has the property that if f is locally distributionally integrable over the real line and $\psi \in \mathcal{D}(\mathbb{R})$ is a test function, then $f\psi$ is distributionally integrable, and the formula

$$\langle \mathsf{f}, \psi \rangle = (\mathfrak{dist}) \int_{-\infty}^{\infty} f(x)\, \psi(x)\, \mathrm{d}x$$

defines a distribution $\mathsf{f} \in \mathcal{D}'(\mathbb{R})$ that has distributional point values almost everywhere and actually $\mathsf{f}(x) = f(x)$ almost everywhere.

The talk is based on collaborative work with R. Estrada [1].

Mathematics Subject Classification 2010: 26A39; 26A24, 26A36, 46F10.

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III Short Communications

Algebra

Some congruences for Fermat quotients

Miomir Andjić[†]
(joint work with Romeo Meštrović[‡])

Let p be a prime, and let a be an integer not divisible by p. Then by Fermat Little Theorem, the difference $a^{p-1}-1$ is divisible by p. The integer

$$q_p(a)=\frac{a^{p-1}-1}{p}$$

is called the Fermat quotient of p to base a.

This quotient has been extensively studied because of its links to numerous question in number theory (see [1], [2] and [4]). It is well known that divisibility of Fermat quotient $q_p(a)$ by p has numerous applications which include the Fermat Last Theorem and squarefreeness testing. Motivated by some results obtained in [3], here we establish some congruences modulo a prime p involving the Fermat quotients $q_p(a)$ with different integer values of a.

Mathematics Subject Classification 2010: 11A07; 05A10, 05A19, 11B65.

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The elements of multiplication lattice modules

Emel Aslankarayiğit Uğurlu[†]
(joint work with Fethi Callialp[‡] and Unsal Tekir[*])

This study concerns with investigation of multiplication lattice modules. We define a new multiplication over elements of a multiplication lattice modules. With this multiplication, we prove some important properties of idempotent element, prime element, weakly prime element and almost prime element in multiplication lattice modules.

Mathematics Subject Classification 2010: 16F410; 16F05.

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Construction of linear codes over $F_q[u,v]/\langle u^q-u, v^q-v\rangle$

Ayşegül Bayram[†]
(joint work with Bahattin Yildiz[‡] and Irfan Siap[†])

In this paper, we study linear and cyclic codes over the non-chain ring
$$F_q[u,v]/\langle u^q-u, v^q-v\rangle$$
where $q=p^r$ is a prime power. A distance preserving Gray map which induces a relation between codes over this ring and $Z_p^{p^2}$ codes is introduced. Further, the algebraic structure of reversible codes over $F_q[u,v]/\langle u^q-u, v^q-v\rangle$ is also studied.

Acknowledgement. This study is supported by The Scientific and Technological Research Council of Turkey (TÜBİTAK) (No: 113F071).

Mathematics Subject Classification 2010: 94B05; 94B15; 11T71.

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Free ternary power-associative groupoids

Vesna Celakoska-Jordanova†
(joint work with Valentina Miovska†)

A ternary groupoid is called power-associative if every mono-generated ternary subgroupoid is a ternary subsemigroup. The class $\mathcal{P}_a$ of ternary power-associative groupoids is a variety. A description of free objects in this variety and their characterization by means of injective ternary groupoids in $\mathcal{P}_a$ are obtained.

Mathematics Subject Classification 2010: 08B20; 03C05

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Description of semireflexive subcategories using the right product

Olga Cerbu[†]
(joint work with Dumitru Botnaru[†])

In this paper we examine the relationships of semireflexive subcategories [2] and the right product of two subcategories [2].

$\mathcal{C}_2\mathcal{V}$ – the category of locally convex topological vector Hausdorff spaces;
$(\mathcal{E}_u, \mathcal{M}_p)$ – (universal epimorphism class, the class of exact monomorphisms);
Π – the subcategory of complete spaces with weak topology;
$\mathcal{S}$ – the subcategory of spaces with weak topology;
Γ_0 – the subcategory of complete spaces;
$q\Gamma_0$ – the subcategory of quasicomplete spaces;
$s\mathcal{R}$ – the subcategory of semireflexive spaces;
$\widetilde{\mathcal{M}}$ – the subcategory of spaces with Mackey topology.

Theorem 1. *Let $(\mathcal{K}, \mathcal{L})$ be a pair of conjugated subcategories* [1] *and Γ is a $\mathcal{M}_p$-reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$, that is $\Gamma \in \mathbb{R}(\mathcal{M}_p)$. Then $\mathcal{L} \times_{sr} \Gamma = \mathcal{K} \times_d (\mathcal{L} \cap \Gamma)$.*

Theorem 2. *Let the right product of the subcategory $\mathcal{K}$ and $\mathcal{R}$ be a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$. The following statements are equivalent:*

1. $\mathcal{K} \times_d \mathcal{R} = \mathcal{R}$.

2. The subcategory $\mathcal{R}$ verifies the conditions $(\mathcal{SR})$ in the relation with the coreflector functor
$k: \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K}$.

Theorem 3. *Let $\mathcal{R} \in \mathbb{R}(\mathcal{E}_u, \mathcal{M}_p)$. The statements are equivalent:*

1. $\widetilde{\mathcal{M}} \times_d \mathcal{R} = \mathcal{R}$.

2. $\mathcal{R}$ is a semireflexive subcategory.

Theorem 4. *Let $\mathcal{R} \in \mathbb{R}(\mathcal{E}_u, \mathcal{M}_p)$. The statements are equivalent:*

1. $\mathcal{K} \times_d \mathcal{R} = \mathcal{R}$, for any coreflective subcategory $\mathcal{K}$ with property $\widetilde{\mathcal{M}} \subset \mathcal{K}$.

2. $\mathcal{R}$ is a semireflexive subcategory.

Examples. 1. *Since $(\widetilde{\mathcal{M}}, \mathcal{S})$ is a pair of conjugated subcategories in the category $\mathcal{C}_2\mathcal{V}$, and $\Pi = \mathcal{S} \cap \Gamma_0$ we have $\mathcal{S} \times_{sr} \Gamma_0 = \Pi = \widetilde{\mathcal{M}} \times_d \Pi$;*

2. *$\mathcal{S} \times_{sr} (q\Gamma_0) = s\mathcal{R} = \widetilde{\mathcal{M}} \times_d (\mathcal{S} \cap q\Gamma_0)$.*

Mathematics Subject Classification 2010: 18A20.

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Second and secondary lattice modules

Fethi Çallialp[†]
(joint work with Unsal Tekir[‡] and Emel A. Uğurlu[‡])

This study concerns with investigation of second and secondary lattice modules. First, we define the concepts of second and secondary module in a lattice module. Next, we give some examples for second and secondary lattice modules. In addition, we show that if M is a multiplication and second lattice module, then M is simple. Finally, we show that if M is a multiplication and secondary lattice module, then M is primary.

Mathematics Subject Classification 2010: 16F410; 16F05.

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Probabilistic groupoid - a new algebraic structure

Lidija Goračinova-Ilieva[†]
(joint work with Smile Markovski[‡])

Algebraic structures are commonly used as a tool in various processes. But their exactness reduces the opportunity of their application in non-deterministic environment. On the other hand, probability theory and fuzzy logic do not provide convenient means for expressing the result of combining elements in order to produce new ones. Moreover, these theories are not developed to "measure" algebraic properties. Therefore, we propose a new concept which relies both on universal algebra and probability theory.

We consider discrete sets with only one binary operation, additionally including the "possibility" of obtaining each particular element as a product. This leads to a structure that we call probabilistic groupoid. "Ordinary" groupoids are just a trivial probabilistic ones.

Let A and B be at most countable non-empty sets, and $\mathcal{D}_B$ is the set of all probability distributions on B. Probabilistic mapping from A to B is a mapping $h : A \to \mathcal{D}_B$.

Let A be a set, $n \in \mathbb{N}$, and $A^n = \{(a_1, a_2, \ldots, a_n) | a_i \in A, i = 1, 2, \ldots, n\}$ is the power-set of A. We define a notion of probabilistic (n -ary) operation on A as a probabilistic mapping from A^n to A. A pair (A, F) of a set A and a family F of probabilistic operations on A is probabilistic algebra. When $F = \{f\}$ has one binary operation, then the probabilistic algebra (A, f) is a probabilistic groupoid. We consider some basic properties of such structures.

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Fiala–Agre list of single axioms for Boolean groups is wrong

Aleksandar Krapež

In their paper 'Shortest single axioms with neutral element for groups of exponent 2 and 3' (Quasigroups and Related Systems 21/1 (2013), 69 − 82) Nick C. Fiala and Keith M. Agre give, among other results, the list of shortest single axioms (in the language $L = \{\cdot, e\}$) for Boolean groups (groups of exponent 2). They explain their choice of 1323 candidate formulas, a series of tests with programs Prover9 and Mace4 they submitted their formulas to and finaly give a theorem in which they list 173 axioms for Boolean groups and further five candidate formulas which are proved to be axioms only for finite models.

Unfortunately, their theorem is wrong. Most of their identities are equivalent to $x = e$ which is an axiom for trivial (one–element) groups. We briefly discuss possible causes of their error.

Mathematics Subject Classification 2010: 20A05.

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Some congruences involving harmonic numbers

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(joint work with Miomir Andjić[‡])

Given positive integers n and m, the harmonic numbers of order m are those rational numbers $H_{n,m}$ defined as

$$H_{n,m} = \sum_{k=1}^{n} \frac{1}{k^m}.$$

If $m = 1$, then $H_n := H_{n,1} = \sum_{k=1}^{n} 1/k$ is the nth harmonic number.

In 2012 Z.-W. Sun [3] obtained basic congruences modulo a prime $p \geq 5$ for several sums of terms involving harmonic numbers. In particular, Sun established $\sum_{k=1}^{p-1}(H_k)^r \pmod{p^{4-r}}$ for $r = 1, 2, 3$. Further generalizations and extensions of these congruences have been obtained by R. Tauraso in [4] and by R. Meštrović in [1] and [2]. Here we prove some new congruences modulo a prime or the square of a prime involving certain numbers $H_{n,m}$ with $n, m \geq 1$.

Mathematics Subject Classification 2010: 11A07; 05A10, 05A19, 11B65.

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Extensions of semicommutative rings

Mohammad Javad Nikmehr

In this paper we introduce the notion of strongly α-semicommutative rings which is a strong version of α-semicommutative rings and investigate their properties. We first give an example to show that strongly semicommutative rings need not be strongly α-semicommutative. We next show for an endomorphism α of a ring R, R is strongly α-semicommutative if and only if $R[x]$ is strongly α-semicommutative if and only if $R[x;x^{-1}]$ is strongly α-semicommutative. Also, for a ring endomorphism α and an α-derivation δ, we introduce the concept of nil (α,δ)-semicommutative rings to investigate the nilpotent elements in semicommutative rings. As a consequence we extend and unify several known results.

Mathematics Subject Classification 2010: 13D02; 13E15, 16E10.

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Linear codes of constant-weight over finite chain rings

Irfan Siap[†]
(joint work with Mustafa Sari[†] and Vedat Şiap[‡])

This paper determines the structure of one-homogeneous weight codes over a finite chain ring and studies the properties of these codes. By taking advantage of a distance-preserving Gray map defined by Jitman and Udomkavanich [1] from R^n to $F_{p^l}^{p^{l(e-1)n}}$ where F_{p^l} is a finite field with p^l elements, R is a finite chain ring and F_{p^l} is the residue field of R, we obtain a family of optimal one-Hamming weight codes over F_{p^l}. Further, we propose a generalized method that also includes the examples of optimal codes obtained by Shi et.al [2].

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Mathematics Subject Classification 2010: 94B05; 94B65.

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An identity between the $m-$spotty weight enumerators of byte linear array codes and its dual

Vedat Şiap[†]
(joint work with Mehmet Özen[‡])

During the process of sending the digital data over communication channel an error in the transmission and/or storage of the digital data may occur that can be detected and/or corrected or simply ignored. They often occur in bursts as in random access memory chips (RAM) that have found applications in computer memory systems. To resolve such a problem spotty and $m-$spotty byte errors are introduced. Here, we present $m-$spotty burst errors and related these families with codes over a special family of Frobenious rings. Further, we apply our results on MacWilliams identity to $m-$spotty array codes.

Mathematics Subject Classification 2010: 94B05; 94B60.

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Strongly 0-dimensional rings and modules

Ünsal Tekir[†]
(joint work with C. Jayaram[‡] and Kürşat Hakan Oral[†])

In this study, a commutative ring R with identity is called *strongly* 0*-dimensional ring* if whenever a prime ideal P of R, contains the intersection of any family of ideals, then P contains one of the ideals of family. Next the following important results are obtained.

Theorem. The following statements on R are equivalent:
(i) R is an Artinian ring.
(ii) R is a strongly 0-dimensional ring and locally Noetherian.
(iii) R is a Noetherian ring and R_P is a strongly 0-dimensional ring for all maximal ideals P of R.

Multiplication R-module M is called strongly 0-dimensional module if whenever a prime submodule N of M, contains the intersection of any family of submodules, then P contains one of the submodules of family. We have the following result.

Theorem. Let M be a faithful multiplication R-module. Consider the following three statements:
(i) M is a von Neumann regular module.
(ii) Every primary submodule of M is a maximal submodule.
(iii) Every primary submodule of M is a minimal prime submodule.

Mathematics Subject Classification 2010: 13A15, 13A99.

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Spec(M) and Separation Axioms Between T_0 and T_1

Gülşen Ulucak [†]
(joint work with Ünsal Tekir[‡])

In this study, we characterize the modules whose prime spectrum satisfy some of the separation axioms between T_0 and T_1. These characterizations are of the notions of pm-module, m-module, ϵ-module, ES-module, D-module and Y-module.

Mathematics Subject Classification 2010: 13C60

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δ-primary submodules of modules

Gürsel Yeşilot

In this study we investigate δ-primary submodules which unify prime submodules and primary submodules. Many results about prime and primary submodules are extended into the this general framework.

Mathematics Subject Classification 2010: 13C13, 13C05.

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On 2-absorbing primary submodules of modules over commutative rings

Ece Yetkin[†]

(joint work with Hojjat Mostafanasab[‡],
Ünsal Tekir[†] and Ahmad Yousefian Darani[‡])

In this work, generalizing 2-absorbing primary ideals of rings, the concept of 2-absorbing primary submodules of modules over commutative rings is defined. Let M be an R-module. A proper submodule N of an R-module M is called a *2-absorbing primary submodule* of M if whenever $a, b \in R$ and $m \in M$ and $abm \in N$, then $am \in M\text{-rad}(N)$ or $bm \in M\text{-rad}(N)$ or $ab \in (N :_R M)$. Some basic properties of 2-absorbing primary submodules are given and also the relations among prime, primary, 2-absorbing and 2-absorbing primary ideals are investigated.

Mathematics Subject Classification 2010: 13A15

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Differential Equations

About compatibility of the Vekua equation with some other equations

Slaǵana Brsakoska

In the paper, the compatibility of the Vekua equation with equations that define some generalized analytical functions will be considered. The main results are formulated in a few theorems.

Mathematics Subject Classification 2010: 34M45

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On the exact solution on a linear differential equation of first order

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(joint work with Boro M. Piperevski[‡] and Elena Hadžieva[*])

We are considering the linear differential equation of first order

$$f(x)y' - f'(x)y = f^3(x)R(x,t), \quad \sqrt{a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0},$$

where $R(x,t)$ is a rational function. We are giving a condition for the equation to have exact solution and a method for finding the solution.

Mathematics Subject Classification 2010: 34A05, 11D25.

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Approximate solution of the Bagley-Torvik equation by using hybridizable discontinuous Galerkin method

Mehmet Fatih Karaaslan[†]
(joint work with Fatih Çeliker[‡] and Muhammet Kurulay[*])

In this study, we use a new and technologic method called as Hybridizable discontinuous Galerkin method for solving Bagley-Torvik equation. Previously, HDG method was not known for solving this model problem. We have seen that it gives us efficient and convergent results. For stability of the system, it is a crucial point to choose stability parameter after we impose the linear system. By using general rule for obtaining stability parameter, we complete our global system. To verify our claim, several numerical examples are presented to demonstrate the accuracy and stability of the HDG method for Bagley-Torvik equation.

Mathematics Subject Classification 2010: 34A08; 65L60.

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Solving of fractional Burger's equation with finite difference method

Muhammet Kurulay[†]
(joint work with Ibrahim Senturk[†])

In this study, we extend the finite difference method to solve nonlinear fractional partial differential equations. The time- and space-fractional Burger's equations with initial and boundary conditions are chosen to illustrate our method. As a result, we successfully obtain some available approximate solutions of them. The results reveal that the proposed method is very effective and simple for obtaining approximate solutions of nonlinear fractional partial differential equations. The fractional derivatives are considered in the Caputo and Riemann-Liouville sense.

Mathematics Subject Classification 2010: 65L12, 58K40.

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Algebraic model of difference equations and functional equations

Jitka Laitochová

We will deal with the theory of Abel functional equations in the space of strictly monotonic functions S. The Abel functional equation model reduces under specialization to a linear functional or a difference equation. Definitions, structure, and general theory for Abel functional equations on S appear. The approach duplicates the rich body of known definitions, results and properties for classical functional and difference equations, see [1], [2].

The setting for the algebraic model is in the space S of strictly monotonic functions f defined on the interval $\mathcal{J} = (-\infty, \infty)$. It is required that f map $\mathcal{J}$ one-to-one onto an interval (a, b), where a and b are real or extended numbers.

The model equation is expressed in terms of iteration of a function Φ in S. The iteration process uses a *canonical function* in S, which is an arbitrarily chosen increasing function $X \in S$.

A method is presented for solving a new model equation. This method can be applied to solve, in particular, some classical linear functional and difference equations, see [3], [4], [5].

Mathematics Subject Classification 2010: 39B05; 39B12.

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Stability and boundedness of solutions of a certain system of third-order nonlinear delay differential equations

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In this paper a number of known results on the stability and boundedness of solutions of some scalar third-order nonlinear delay differential equations are extended to some vector third-order nonlinear delay differential equations.

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On a Shapkarev's method of differentiation and transformation

Boro M. Piperevski[†]
(joint work with Elena Hadžhieva[‡])

This paper is to point out the contribution of Prof. Dr. Ilija A. Shapkarev in the field of differential equations, particularly in existence and construction of polynomial solution of the ordinary differential equation of n-th order, or equivalently, of system of n equations of first order. The method that he uses is called method of differentiation and transformation (MDT). With this paper we give a review of his work and extension of the application of his method.

Mathematics Subject Classification 2010: 34A05, 34C20.

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Monotone iterative technique for the initial value problem for delay differential equations with multi-point delay jump conditions

Kremena Stefanova[†]
(joint work with Snezhana Hristova[†])

An initial value problem for nonlinear impulsive delay differential equations with jump conditions at several delay time points is studied. An algorithm based on the monotone iterative technique for constructing successive approximations of the solution of the considered problem is given. Two monotone increasing and decreasing sequences are constructed and it is proved both approaching to the solution of the given problem. Every successive approximation is the unique solution of an appropriately chosen initial value problem for a linear impulsive delay differential equation with multi-point delay jump conditions. A formula for the solution of these linear equations is obtained. A comparison result is proved. Also, every successive approximation is a lower/upper solution of the given problem.

Mathematics Subject Classification 2010: 34A37, 34A12, 34A45.

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Geometry and Differential Geometry

Explicit solutions of the natural PDE's of minimal surfaces

Georgi Ganchev

We consider minimal surfaces in the four-dimensional Euclidean space and space-like or time-like minimal surfaces in the four-dimensional Minkowski space. We study the natural PDE's of these minimal surfaces with respect to natural parameters. We give a description of the solutions of the natural equations of minimal surfaces in the four-dimensional Euclidean space.

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Curvature inheritance symmetry

Chayan Kumar Mishra

In this paper we study curvature inheritance symmetry and Ricci-Inheriting symmetry in Finsler space and investigated some results.

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Holomorphic curvatures of twistor spaces

Oleg Mushkarov[†]
(joint work with Johann Davidov[†])

We study the twistor spaces of oriented Riemannian four-manifolds as a source of almost Hermitian six-manifolds of constant or strictly positive holomorphic, Hermitian and orthogonal bisectional curvatures. In particular, we obtain explicit formulas for these curvatures in the case when the base manifold is Einstein and self-dual, and observe that the "squashed" metric on $\mathbb{CP}^3$ is a non-Kähler Hermitian-Einstein metric of positive holomorphic bisectional curvature. This shows that a recent result of Kalafat and Koca in dimension four cannot be extended to higher dimensions. We prove that the Hermitian bisectional curvature of a non-Kähler Hermitian manifold is never a non-zero constant which gives a partial negative answer to a question of Balas and Gauduchon.

Mathematics Subject Classification 2010: 53C28, 53C21

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On the Quantum Supergroup $U_q(osp(1,2))$

Muttalip Özavşar[†]
(joint work with Giovanni Landi[‡])

Zou constructed a dual Hopf superalgebra to the quantum supergroup $U_q(osp(1,2))$ [1]. In this presentation, we first construct an involution algebra for $U_q(osp(1,2))$ in such a way that the duality is compatible with the involution structures in the related Hopf superalgebras, and by using this construction, we construct the related left(right) $U_q(osp(1,2))$- module *-superalgebras.

Mathematics Subject Classification 2010: 16T05, 81R50, 81R60.

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On the linearly independent vector fields on Grassmann manifolds

Kostadin Trenčevski

In this paper are found $\theta(n)$ linearly independent vector fields on the Grassmann manifold $G_k(V)$ of k-planes in n-dimensional Euclidean vector space if k is odd number, where $\theta(n)$ is the maximal number of linearly independent vector fields on S^{n-1}, i.e. skewsymmetric anticommuting complex structures on $\mathbb{R}^n$.

Mathematics Subject Classification 2010: 53C30.

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Numerical Optimization and Approximations

Speed control in numeric controlled systems

Igor Dimovski[†]
(joint work with Samoil Samak[‡], Dijana Cvetkoska[‡],
Mirjana Trompeska[‡] and Filip Kočoski[‡])

In order to achieve higher speed (higher productivity at the same time), the modern way of managing numerical controlled systems includes Look Ahead algorithms with strong mathematical background. The purpose of these algorithms is generating a speed profile with which the tool will move along the programmed movement path.

In this article will be described a method for speed profile generating whereby we will use numerical methods for differential computing, spline interpolation/approximation and linear programming. For testing and view of the generated speed profiles we will use the programming package MATLAB.

Mathematics Subject Classification 2010: 93C85

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Stability of difference schemes for a fractional Schrödinger differential equation

Betul Hicdurmaz[†]
(joint work with Allaberen Ashyralyev[‡])

A summary of the obtained results by the authors for the fractional Schrödinger differential equations is presented ([1], [2]). Some new difference schemes are constructed for a fractional Schrödinger differential equation. New stability estimates are obtained for second order of accuracy difference schemes for the fractional Schrödinger differential equation. Theoretical results are supported by the numerical experiments.

Mathematics Subject Classification 2010: 35R11; 65M06.

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A line search method with memory for unconstrained optimization of noisy functions

Filip Nikolovski[†]
(joint work with Nataša Krejić[‡], Zorana Lužanin[‡] and Irena Stojkovska[†])

We propose a new line search method for unconstrained optimization of noisy functions. The nonmonotone line search rule is based on Ulbrich's nonmonotone component ([1]). The method uses only nosy functional values. Convergence under standard assumptions is established. Computational results show a good performance of the method compared with the monotone one.

Mathematics Subject Classification 2010: 90C56, 65K05.

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Performance of descent stochastic line search methods for unconstrained optimization in noisy environment

Irena Stojkovska[†]
(joint work with Nataša Krejić[‡], Zorana Lužanin[‡] and Zoran Ovcin[*])

We explore performance of new descent stochastic line search method (DSLS) for unconstrained noisy optimization. DSLS method is a combination between a line search method with an arbitrary descent direction in initial phase when iterates are far from the solution and stochastic approximation (SA) afterwards. Negative gradient, BFGS and SR1 directions are tested, and different noise levels are considered. Algorithm used for practical implementation successfully overcomes several issues originated from the presence of noise. Tests performed on a test collection of 44 problems, showed a good performance of DSLS methods, regarding different performance measures.

Mathematics Subject Classification 2010: 65K05, 62L20.

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Barzilai-Borwein method for a nonlocal elliptic problem

Todor Todorov

The object of interest in the present paper is a nonlocal nonlinear second order elliptic problem arising from the population biology. The nonlocal term involved in the strong problem essentially increases the complexity of the problem and the necessary total computational work. The nonlinear weak formulation of the problem is reduced to minimization of a nonlinear functional. Finite element discretizations by Lagrangian finite elements are applied for obtaining an approximate discrete minimization problem. Due to its simplicity and efficiency, the Barzilai and Borwein (BB) gradient method is used for finding positive solutions with respect to the inhomogeneous strong Allee effect growth pattern. The corresponding fast and stable iterative algorithm is described step by step. The new method is computer implemented and the results of the test examples are presented graphically.

Mathematics Subject Classification 2010: 65N30.

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Probability and Statistics

Influence of the latent trait distribution shape to the expected item score

Dimitar Atanasov

There is a strong assumption under the Item Response Theory (IRT), that the latent trait follows the Normal distribution in the observed population or sample. Under this assumption the expected probability for the correct item performance can be expressed as by the IRT parameters of the items (as well as many other item and test characteristics). Violation of this assumption is very common in everyday practice of test evaluation.

In this work a simulation study of the robustness of the expected probability for the correct item performance is presented. To model the deviation of the distribution of the population ability from the normal distribution, the g-and-h distribution is used.

Mathematics Subject Classification 2010: 91E10; 62P15.

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Randomly indexed central order statistics

Aneta Gacovska-Barandovska[†]
(joint work with Elisaveta I. Pancheva[‡])

In [1] we have considered the upper order statistics with central rank of sample with deterministic size. Here we investigate the asymptotic behavior of randomly indexed upper order statistics using regular norming time-space changes.

Mathematics Subject Classification 2010: 62G20, 62G30, 62E20.

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On the bivariate inflated-parameter generalized power series distributions

Krasimira Kostadinova[†]
(joint work with Leda Minkova[‡])

The family of Inflated–parameter Generalized Power Series distributions (IGPSD) was introduced by Minkova in 2002 as a compound Generalized Power Series distributions (GPSD) with geometric compounding distribution. In these notes we introduce a family of compound GPSDs with bivariate geometric compounding distribution. The probability mass function, recursion formulas, conditional distributions and some properties are given. A member of this family is a Type II bivariate Pólya-Aeppli distribution, introduced by Minkova and Balakrishanan (2014). In this notes the particular cases of bivariate compound binomial, negative binomial and logarithmic series distributions are analyzed in detail.

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Mathematics Subject Classification 2010: 60E05.

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Type III bivariate compound generalized power series distributions

Meglena Lazarova[†]
(joint work with Leda Minkova[‡])

The family of Inflated–parameter Generalized Power Series distributions (IGPSD) was introduced by Minkova in 2002 as a compound Generalized Power Series distributions (GPSD) with geometric compounding distribution. In these notes we start with a bivariate power series distributions and compound it with independent geometrically distributed random variables. The probability mass function, recursion formulas, conditional distributions and some properties are given. The particular cases of bivariate compound binomial, negative binomial and logarithmic series distributions are analyzed in detail. The case of compound bivariate Poisson distribution simplifies to independency.

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Mathematics Subject Classification 2010: 60E05.

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McDonald Lindley distribution: Properties and Applications

Faton Merovci

In this paper, we present a new class of distributions called McDonald Lindley distribution. This class of distributions contains several distributions such as beta-Lindley, Kumaraswamy-Lindley and Lindley as special cases. The hazard function, reverse hazard function, moments and mean residual life function are obtained. We estimate the parameters by maximum likelihood and provide the observed information matrix. The usefulness of the new distribution is illustrated with real data set that show that it is quite flexible in analyzing positive data instead of the McDonald Lindley distribution and Lindley distributions.

Mathematics Subject Classification 2010: 62N05, 90B25.

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Modeling and estimating multitype branching processes with negative multinomial offspring distributions

Vessela Stoimenova [†]
(joint work with Ana Staneva[‡])

We consider multitype branching stochastic processes with offspring distributions from the negative multinomial distribution family. We present some possible models for applications and introduce the Bayesian inference in two sampling schemes - when the entire family tree is observed and when observations only on the generation sizes are made. In the special case of a two-type branching process with negative binomial offspring distribution we use a randomized algorithm to switch from the generation sizes scheme to family tree observations. We derive a lower bound of the breakdownpoint of the trimmed likelihood - a robust modificaion of the maximum likelihood, appropriate for estimaton in a contaminated sample. The applicability and adequacy of the robust estimation in the presence of outliers is shown via simulations and computational results.

Mathematics Subject Classification 2010: 60J80; 62F15, 62F35.

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Combinatorial analysis and applications of experimental design

Petya Valcheva

This article describes different types of combinatorial configurations such as Balanced incomplete block designs (BIBDs), Orthogonal arrays (OAs) and Latin squares (LSs). We demonstrate some illustrative examples that show the application of these designs in the Experimental Designs. It is shown an important application in error-correcting code theory.

Acknowledgement. This work was supported by the European Social Fund through the Human Resource Development Operational Programme under contract BG051PO001-3.3.06-0052 (2012/2014)

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Some sufficient conditions for univalence depending on two parameters

Edmond Aliaga[†]
(joint work with Nikola Tuneski[‡])

This paper is a continuation of previous work done by D. Aharonov and U. Elias. New criteria (sufficient conditions) for univalence of some analytic functions with two parameteres will be presented. The proofs involve Shwartzian derivative and Nehari's theorem.

Mathematics Subject Classification 2010: 30C45.

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Weighted Norlund-Euler A-statistical convergence for sequences of positive linear operators

Ekrem Aljimi[†]
(joint work with Elida Hoxha[†] and Valdete Loku[‡])

We introduce the notion of weighted Norlund-Euler A-statistical convergence of a sequence, where A represents the nonnegative regular matrix. We also prove the Korovkin approximation theorem by using the notion of weighted Norlund-Euler A-statistical convergence. Further, we give a rate of weighted Norlund-Euler A-statistical convergence.

Mathematics Subject Classification 2010: 40G15; 41A10; 40A05, 40C05.

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Some Fixed Point Results in Convex Metric Spaces

Yunus Atalan[‡]
(joint work with Vatan Karakaya[‡] and Faik Gürsoy[*])

In this presentation, we give strong convergence and stability results for two-step iterative process in convex metric spaces by using nonlinear mappings.

Mathematics Subject Classification 2010: 47H10.

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Sandwich-type subordinations and superordinations for analytic integral operators

Teodor Bulboacă[†]

The lecture deals with some recent results of the author about different classes of analytic integral operators that preserve the subordination and the superordination, in order to obtain *sandwich-type* and *modified* (*weighted*)*sandwich-type* results. Applications obtained for appropriate choices of the parameters are also given.

Mathematics Subject Classification 2010: 80C80; 30A20, 30A40.

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On some fixed point theorems for a family of mappings of Perov type

Marija Cvetković[†]
(joint work with Vladimir Rakočević[†], Ljiljana Gajić[‡] and Dejan Ilić[†])

In this lecture we will be talking about some fixed point results of a family of mappings on cone metric space satisfying some generalized contractive conditions including positive operator with spectral radius less than 1. The results for normal cone metric space will be also presented. The main results could not be derived from analogous metric space theorems by the scalarization method, and hence indeed improves various results on cone metric spaces.

Mathematics Subject Classification 2010: 47H10; 54H25.

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Two step perturbed iterative algorithm for generalized nonlinear implicit quasi-variational inclusion

Kadri Doğan[†]
(joint work with Vatan Karakaya[†])

In this presentation, we study a class of generalized nonlinear implicit quasi-variational inclusions and prove its equivalence with a class of fixed point problems by making use of the properties of maximal monotone. We also prove the existence of solutions for this generalized nonlinear implicit quasi-variational inclusions and the convergence of iterative sequences generated by the perturbed algorithms.

Mathematics Subject Classification 2010: 47H10.

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The hybrid weighted diaphony

Vasil Grozdanov

The hybrid sequences and nets are objects that combine the advantages of sequences and nets with rational and irrational coordinates of their own points. In our talk we will present a new hybrid version of the diaphony, the so-called hybrid weighted diaphony.

We will consider a hybrid orthonormal function system which is a tensor product of the trigonometric system, the system of Walsh functions over finite groups, the b-adic arithmetic function system and Vilenkin function system.

By using this function system we introduce a hybrid version of the weighted diaphony and prove that this kind of the diaphony is a quantitative measure for the irregularity of the distribution of sequences.

We introduce a hybrid reproducing kernel Hilbert space which is based on using the hybrid function system. A formula in explicit form for the worst-case error of the integration in the considered Hilbert space is obtained. The worst-case error of the integration in this space and the hybrid diaphony are connected.

Mathematics Subject Classification 2010: 65C05, 65C20, 11K36

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Faster iterative algorithms for general variational inequalities and nonexpansive mappings

Faik Gürsoy

In this presentation the following efficient iterative algorithms are introduced for finding solutions of a particular problem, which was earlier proposed and studied by Noor [1]:

$$\begin{cases} w_{n+1} = S\left\{v_n - g\left(v_n\right) + P_H\left[g\left(v_n\right) - \sigma T v_n\right]\right\}, \\ v_n = \left(1-\xi_n^1\right) S\left\{w_n - g\left(w_n\right) + P_H\left[g\left(w_n\right) - \sigma T w_n\right]\right\} \\ \qquad +\xi_n^1 S\left\{\varpi_n - g\left(\varpi_n\right) + P_H\left[g\left(\varpi_n\right) - \sigma T \varpi_n\right]\right\}, \\ \varpi_n = \left(1-\xi_n^2\right) w_n + \xi_n^2 S\left\{w_n - g\left(w_n\right) + P_H\left[g\left(w_n\right) - \sigma T w_n\right]\right\}, \end{cases} \tag{1}$$

$$\begin{cases} p_{n+1} = S\left\{q_n - g\left(q_n\right) + P_H\left[g\left(q_n\right) - \sigma T q_n\right]\right\}, \\ q_n = \left(1-\xi_n\right) p_n + \xi_n S\left\{p_n - g\left(p_n\right) + P_H\left[g\left(p_n\right) - \sigma T p_n\right]\right\}, \end{cases} \tag{2}$$

where $\mathcal{H}$ is a real Hilbert space, $H \subset \mathcal{H}$ is a nonempty closed convex set, T, $g : H \to \mathcal{H}$ are two nonlinear operators, $S : H \to H$ is a nonexpansive mapping and $\{\xi_n\}_{n=0}^{\infty}$, $\{\xi_n^0\}_{n=0}^{\infty}$, $\{\xi_n^1\}_{n=0}^{\infty}$, $\{\xi_n^2\}_{n=0}^{\infty} \subset [0,1]$ real sequences satisfying certain control condition(s). It has been shown that these iterative algorithms converges strongly to the solutions of the problem considered in [1]. Further, it has been established equivalence of the convergence among the new iteartive algorithms and some other iterative algorithms introduced in [1]. Finally, It has been shown that these new iterative algorithms converges at rate faster than the iterative algorithms in [1].

Mathematics Subject Classification 2010: 49J40, 58E35, 47H10.

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Nonlinear contractions and fixed points in complete b-dislocated metric spaces

Elida Hoxha[‡]
(joint work with Kastriot Zoto[†] and Panda Sumati Kumari[*])

In this paper, we continue the study of complete dislocated and b-dislocated metric spaces and established some common fixed point theorems for two and four mappings. Our results generalizes and extend some existing results in the literature in a class effectively larger such as b-dislocated metric spaces, where the self distance for a point may not be equal to zero.

Mathematics Subject Classification 2010: 47H10; 54H25.

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Generalizations of Steffensen's inequality by Hermite's polynomial

Julije Jakšetić[†]
(joint work with Josip Pečarić[‡] and Anamarija Perušić[*])

We study generalizations of Steffensen's inequality using Hermite expansions with integral reminder. In comparing differences of two weighted integrals we vary on the number of knots in expansion which leads us to generalization of conditions for Steffensen's inequality. After that, we construct associated exponentially convex functions and Cauchy means.

Mathematics Subject Classification 2010: 26D15, 26D20.

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Distributions, neutrix calculus and special functions

Biljana Jolevska-Tuneska

In this paper we use the neutrix calculus to extend the definition on some special functions for the negative integers.

Mathematics Subject Classification 2010: 46F10.

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Tauberian results for the short-time Fourier transform of exponential distributions

Sanja Kostadinova Atanasova[†]
(joint work with Stevan Pilipović[‡], Katerina Saneva[†] and Jasson Vindas[*])

We study the short-time Fourier transform on the space of distributions of exponential type. We also obtain various Tauberian theorems for the short-time Fourier transform.

Mathematics Subject Classification 2010: 81S30, 40E05; 26A12, 41A60, 46F05, 46F12.

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Real earth based spline for gravitational potential determination

Elena Kotevska[†]
(joint work with Willi Freeden[‡])

For computational reasons, the spline interpolation of the Earth's gravitational potential is usually done in a spherical framework [3]. However, the increasing observational accuracy require adequate mathematical methods and observing of the geophysically more relevant surfaces. In this work, we propose a spline method with respect to the real Earth surface. The spline formulation reflects the specific geometry of a given regular surface. This is due to the representation of the reproducing kernel as a Newton integral over the inner space of a regular surface. The approximating potential functions have the same domain of harmonicity as the actual Earth's gravitational potential. This is a step forward in comparison to the spherical harmonic spline formulation involving functions harmonic down to the Runge sphere. Moreover, this kernel represents a generalization to spherically oriented kernels.

Mathematics Subject Classification 2010: 31B20; 31B10, 31B05.

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More precise weak majorization relations for the Jensen inequality and applications

Mario Krnić [†]
(joint work with Josip Pečarić[‡])

Motivated by results of Aujla and Silva ([1]), in this talk we give several more precise weak majorization and eigenvalue inequalities for some matrix versions of the famous Jensen inequality with regard to a convexity. Our main results are then applied to log convex functions. As an application, we obtain refinements of some important matrix inequalities known from the literature.

Mathematics Subject Classification 2010: 26A51, 47A63; 15A42, 47B15.

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Probability distributions associated with one generalization of the Planck's law

Delčo Leškovski

The main object of this talk is to present a study of probability density functions and distributions associated with a generalized Planck's law. Characteristic functions and fractional moments related to the probability density functions of the considered distributions are derived by means of generalized Hurwitz-Lerch Zeta function introduced by Goyal and Laddha.

Mathematics Subject Classification 2010: 33B15, 33E20, 60E10.

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Hahn-Banach theorem for cyclic 2-subspaces

Aleksa Malčeski

An extension of 2-skew symmetric form defined on a specific kind of 2-subspaces of a 2-vector space, namely, on a 2-subspace with a generating set

$$\{(x_1, x_2), (x_2, x_3), \ldots, (x_{n-1}, x_n), (x_n, x_1)\}, \text{ for } n > 3,$$

is considered. An extension of 2-skew symmetric form defined on a cyclic 2-subspace in the sense of Hahn-Banach is considered as well.

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Hahn-Banach Theorem for branch 2-subspaces

Aleksa Malčeski[†]
(joint work with Vesna Manova-Erakoviḱ[‡])

An extension of 2-skew symmetric form defined on a branch 2-subspace and extension of 2-skew symmetric form defined on a cyclic 2-subspace in the sense of Hahn-Banach is considered. An extension on two types of branch subspaces, namely

$$\{(x_1, x_2), (x_2, x_3), \dots, (x_n, x_{n+1}), \dots\} \text{ and}$$
$$\{\dots, (x_{-(n+1)}, x_{-n}), \dots, (x_{-2}, x_{-1}), (x_{-1}, x_1), (x_1, x_2) \dots, (x_n, x_{n+1}), \dots\}$$

is considered as well.

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About the 2-Banach spaces

Risto Malčeski[†]
(joint work with Katerina Anevska[†])

In this paper a few properties about convergent sequences into a real 2-normed space $(L, \| \cdot, \cdot \|)$ and into a 2-pre-Hilbert space $(L, (\cdot, \cdot|\cdot))$, that are actually a generalization of appropriate properties of convergent sequences into a pre-Hilbert space are proved. Two characterizations of 2-Banach spaces are given as well. These characterizations are in fact a generalization of appropriate results in Banach spaces.

Mathematics Subject Classification 2010: 46B20; 46C05.

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Example of a holomorphic function on a unit disc without radial limits

Ljupčo Nastovski[†]
(joint work with Petar Sokoloski[†] and Biljana Načevska[‡])

A construction of a holomorphic function f on a unit disc is given, so that $f(re^{i\theta}) < g(r)$, for every $r \in [0,1)$, $\theta \in [0,2\pi)$, where $g : [0,1) \to \mathbb{R}$ is a monotone increasing function such that $\lim_{r\to 1} g(r) = \infty$ and f does not have a radial limit in every $e^{i\theta}$.

Mathematics Subject Classification 2010: 32A40.

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Generalizations of Steffensen's inequality via n weight functions

Anamarija Perušić Pribanić[†]
(joint work with Andrea Aglić Aljinović[‡] and Josip Pečarić[*])

In this talk I will present the new generalizations of Steffensen's inequality, obtained by means of weighted Montgomery identity with n different weighted functions. Instead of holding for a nonincreasing (1-convex) function our generalizations also hold for a n-convex function. Furthermore, we will observe the functionals associated to these new generalizations and use them to construct exponentially convex functions and Cauchy means.

Mathematics Subject Classification 2010: 26D15, 26A48.

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Generalization of Steffensen's inequality by way of two-point Taylor expansion

Marjan Praljak[†]
(joint work with Asfand Fahad[‡] and Josip Pečarić[*])

We use Faà di Bruno's formula for higher order derivatives of the composition of functions and the two-point Taylor expansion with the remainder given in the integral form to derive integral identities. We study sufficient conditions for the positivity of the kernel of the integral remainder and, as a consequence, derive new inequalities. In particular, a generalization of Steffensen's inequality is obtained.

Mathematics Subject Classification 2010: 26D15; 26D10.

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Convergence of some special harmonic functions

Vasko Rečkovski[†]
(joint work with Vesna Manova-Erakovik[‡] and Nikola Rečkovski[*])

In this paper we consider one problem of convergence of sequence of harmonic functions that are important in the theory of the representation of distributions as boundary values of analytic functions.

Mathematics Subject Classification 2010: 44A15, 46F12, 46F20.

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Gauss-Steffensen type inequalities

Ksenija Smoljak Kalamir[†]
(joint work with Josip Pečarić[†])

In 1982 Pečarić obtained Gauss-Steffensen's inequality which reads:

Let $G : [a, b] \to \mathbb{R}$ be an increasing function and $f : I \to \mathbb{R}$ be a nonincreasing function (I is an interval from $\mathbb{R}$ such that $a, b, G(a), G(b) \in I$). If $G(x) \geq x$, then

$$\int_a^b f(x)G'(x)dx \geq \int_{G(a)}^{G(b)} f(x)dx. \tag{1}$$

If $G(x) \leq x$, the reverse inequality in (1) *is valid.*

Using Gauss-Steffensen's inequality we derive generalized Gauss-Steffensen type inequalities related to the class of functions that are "convex at point c". Moreover, as a consequence we obtain inequalities involving the class of convex functions.

Furthermore, we apply obtained inequalities to construct new Stolarsky type means.

Mathematics Subject Classification 2010: 26D15; 26A51.

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Wave fronts of ultradistributions via Fourier series coefficients

Petar Sokoloski

We shall use the properties of the product of periodic ultradistributions and give a new description of the wave front of an ultradistribution $f \in (\mathcal{D}^*)'(\mathbb{R}^d)$ in terms of Fourier series coefficients.

Mathematics Subject Classification 2010: 35A18; 46T30,46F30.

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On limit and derivation of composition

Nikita Shekutkovski

A notion of derivation is presented for a wider class of subsets of the real line. For this generalized situation, by use of several theorems for limits, it is presented a new proof of the chain rule and inverse function theorem for derivation.

Mathematics Subject Classification 2010: 26A03.

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Some results on two classes univalent functions

Nikola Tuneski[†]
(joint work with Edmond Aliaga[‡])

Let $\mathcal{A}$ be the class of function that are analytic in the unit disk $\mathbb{D}$ and are normalized such that $f(z) = z + a_2 z^2 + \cdots$. A function f from $\mathcal{A}$ is said to be α-convex of Janowski type if

$$(1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right) \prec \frac{1+Az}{1+Bz},$$

where $-1 \leq B \leq A \leq 1$, $\alpha \in \mathbb{R}$ and "$\prec$" denotes the usual subordination. Here, using methods from the theory of differential subordinations, we give several results describing the relation between the class of α-convex Janowski type functions and the class

$$\mathcal{U}(\lambda,\mu) = \left\{ f \in \mathcal{A} : \frac{z}{f(z)} \neq 0 \text{ and } \left| \left(\frac{z}{f(z)}\right)^{1+\mu} \cdot f'(z) - 1 \right| < \lambda,\ z \in \mathbb{D} \right\},$$

$\mu \in \mathbb{C}$ and $\lambda > 0$.

Mathematics Subject Classification 2010: 30C45.

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On the exponential ultradistribution semigroups

Daniel Velinov

The exponential ultradistribution semigroups defined by M.Kostić and S. Pilipović, are considered. The structure of the exponential ultradistribution semigroups is given as an addition of the already known structure theorem and some properties of the exponential ultradistribution semigroups are given. Also notation of holomorphic semigroups is generalized to holomorphic exponential ultradistribution semigroups.

Mathematics Subject Classification 2010: 47D03, 47D06; 47D60, 46F20.

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Some inequalities for the Čebyšev functional and general four-point quadrature formulae of Euler type

Ana Vukelić[†]
(joint work with Milica Klaričić Bakula[‡], Josip Pečarić[*] and Mihaela Ribičić Penava[**])

We use inequalities for the Čebyšev functional in terms of the first derivative (see [1]), for some new bounds for the remainder of four-point quadrature formulae of Euler type and its generalizations for Bullen-Simpson's 3/8 formula. As special cases, we consider some new bounds for Euler Simpson's 3/8 formula, Euler-Simpson's formula and Gauss 2-point formula.

Mathematics Subject Classification 2010: 26D15; 26D20, 26D99.

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On the convergence and data dependence results for multistep picard-mann iteration process in the class of contractive-like operators

Isa Yildirim[†]
(joint work with Nazli Kadioglu[†])

In this work, we introduce a new iteration process and we show that this iteration process can be used to approximate fixed point of contractive-like operators. We also prove some data dependence results for contractive-like operators by using this iteration process.

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On boundedness of fractional maximal operator in the weighted $L^{p(\cdot)}(0,1)$ space

Yusuf Zeren

In the last century, several changes of mechanical structures have been discovered in a private environment. In this project, which is close to their concrete problems linking species been investigated the effects of natural environments and fluids processes shape the conditions of growth of non-standard types of non-linear parabolic and elliptic partial differential equations leads to the examination. There are solutions to the equations of this type are collected in the natural index variable type space $L_{p(x)}$.

A lot of the characteristics of these spaces, for example, the structure of joint space, reflection, smooth functions wherever compactness, continuity and compact embedding question adequately studied.

We prove a new sufficiency result for the two weighted boundedness of the fractional maximal operator in the variable exponent Lebesgue space $L^{p(\cdot)}(0,1)$. A complete analog of the SawyerâĂŹs condition are obtained on the weight functions assuming a usual log-Hölder continuity condition on the exponent function $p(.)$.

Mathematics Subject Classification 2010: 42B20; 42B25; 42B35.

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Generalized convex sets in complex and hypercomplex analysis

Yuri Zelinskii

1. **The sphere problem.** Is there a linearly convex compact in two-dimensional complex space $\mathbb{C}^2$, for which all cohomology groups coincide with the corresponding cohomology group of two-dimensional sphere S^2?

2. The questions to approximations of linearly convex sets by sets of the same class, but with smooth or nearly smooth boundary.

3. The description of strong linearly convex compacts by their extreme and foreseeable points of the boundary.

4. **The shadow problem.** What is minimal number of pair wise disjoint ball with the centre on sphere S^{n-1} it is enough that any straight line, getting through the centre of the sphere, crossed at least one of these balls?

5. A characterization of curves and surfaces by some properties of their intersection with algebraic curves and surface of the fixed order.

6. The description hyperspace of linearly convex sets and thick subset in him. Here are desired effects, look like studies L. Bazylevych, but main difficulty of the complex case, unlike to real analysis, is in nonlinear structure of such hyperspaces.

Some of the problems of this theme are connected with the known Ulam problem from Scottish book.

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Section: Topology and Dynamical Systems

Proximate ray. Ends

Beti Andonovikj*
(joint work with Nikita Shekutkovski‡ and Tatjana Atanasova-Pachemska†)

For non-compact spaces the notion of proximate ray will be introduced and will be presented interactions with space of ends (Freudenthal ends).

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Some properties of metrizable LC^n-spaces

Zenafer Bajmak

We are going to establish some properties of metric LC^n-spaces using the corresponding properties of metric ANR-spaces.

Theorem 1. (Vesko Valov) *Every Tychonoff space M is the image of a Tychonoff space X with $\dim X \leq n$ under a perfect n-invertible map. In case M is metrizable, X can be supposed to be also a metrizable space with $w(X) = w(M)$.*

Theorem 2. (Vesko Valov) *Let M be a metrizable LC^n-space and α an open cover of M. Then there exists an open cover β of M refining α such that for any two β-near maps $f, g\colon Z \to M$ defined on a metrizable space Z of dimension $\leq n$ any β-homotopy $H\colon A \times [0,1] \to M$ between $f|A$ and $g|A$, where A is closed in Z, can be extended to an α-homotopy $\widetilde{H}\colon Z \times [0,1] \to M$ connecting f and g.*

Using Theorem 1 and Theorem 2, we prove some properties for metrizable LC^n-spaces.

Mathematics Subject Classification 2010: 54F45, 54C55.

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Quasitoric manifolds and small covers over n-colored simple polytopes

Đorđe Baralić

Quasitoric manifolds and small covers are manifolds such that the orbit space of certain action of torus is a simple polytope. They are topological generalizations of toric varieties from algebraic geometry and central objects of toric topology and toric geometry. Studying of this manifolds is motivated by their numerous applications in topology, combinatorics, robotics and etc. Combinatorial properties of underlying simple polytope play an important rule in the description of the cohomology ring and the characteristic classes of these manifolds. In this contribution, we study a wide class of n-dimensional simple polytopes that posses a coloring of their facets in exactly n colors. This purely combinatorial property highly affects topology and geometry of this manifolds such as immersion and embeddings heights, cobordism rings, etc.

Mathematics Subject Classification 2010: 57N35, 52B12.

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Separation properties of $(3, 1, \rho)$-metrizable spaces and $(3, 2, \rho)$-metrizable spaces

Sonja Čalamani[†]
(joint work with Dončo Dimovski[‡])

For a given $(3, j, \rho)$-metric d on a set M, $j \in \{1, 2\}$, we examine some separation properties of $(3, j, \rho)$-metrizable spaces and show that the separation axioms in $(3, j, \rho)$-metrizable spaces are valid if ρ satisfies some additional conditions.

Mathematics Subject Classification 2010: 54A10, 54E35, 54E99.

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Geometry of Spin Manifolds

Dončo Dimovski[†]

We generalize the well known geometric characterizations of orientable n-dimensional manifolds, i.e. an n-dimensional closed PL manifold M is orientable if and only F in M has a a regular neighborhood homeomorphic to the product of the surface F with an $(n-2)$ ball, and each embedded closed nonorientable surface F in M has a a regular neighborhood homeomorphic to the neighbourhood of F in the Euclidean space E^n.

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Equivalence of intrinsic shape based on V-continuous functions and shape

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(joint work with Nikita Shekutkovski[†], Gjorgji Markoski[†] and Martin Shoptrajanov[†])

Equivalence of intrinsic shape and shape is indirectly proven in [5], using [3] and [4].

In this paper an other, a direct proof is given, which shows that the intrinsic shape constructed with V-continuous functions by Shekutkovski ([2]) is equivalent to external shape constructed with continuous functions ([1]). The established functor, maps from the classes of fundamental sequences to the classes of proxsimative sequences.

Mathematics Subject Classification 2010: 54C56

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For a class of autonomous dynamical systems

Boro M. Piperevski

This paper considers a class of autonomous dynamical systems in many aspects. It shows the connection with EPGLT and EPGD [1], associated with three type's diffeomorphism. It studied attractor (attractors) for the solution (solutions) at some special autonomous dynamical systems classes. By that, it shows some important properties from topological and metric aspect.

Mathematics Subject Classification 2010: 37G35, 34C23, 34K18.

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Intrinsic shape

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In the last decade, intrinsic shape is one of the main fields of mathematical research at University of St. Cyril and Methodius, Skopje. The intrinsic approach to shape will be presented based on proximate sequences and nets of functions, equivalence of different definitions, and recently obtained results. Among them, a non-compact analogue of Borsuk theorem for components, construction of strong shape, of proximate fundamental group and higher proximate groups - invariants of pointed intrinsic shape, and applications to dynamical systems.

Mathematics Subject Classification 2010: 54C56.

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Inclusion induces strong shape equivalence from a global attractor to its compact phase space

Martin Shoptrajanov[†]
(joint work with Nikita Shekutkovski[†])

The first intrinsic approach to shape is given in the papers from J.M.Sanjurjo and J. Felt. In the paper from N.Shekutkovski for the first time the construction of the strong shape category SSh for compact metric spaces is given, using the intrinsic approach. The approach combines continuity up to a covering and the corresponding homotopies of second order. We shall present the strong shape version result for global attractors in compact metric spaces using this approach.

Mathematics Subject Classification 2010: 54H20, 54C56, 37B25

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Higher order proximate groups

Aneta Velkoska[†]
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Using the intrinsic definition of shape based on proximate sequences for compact and paracompact spaces based on proximate nets indexed by open coverings in the paper [1] we defined proximate fundamental group, an invariant of pointed shape of a space.

In this paper, the higher order proximate groups are introduced. It will be shown that these groups are invariants of pointed intrinsic shape.

Mathematics Subject Classification 2010: 54H20, 54C56, 37B25

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The use of fuzzy soft matrices on fuzzy soft multisets in an optimal decision process

Arzu Erdem[†]
(joint work with Çiğdem Gündüz Aras[†])

In this paper, we introduce a concept of a fuzzy soft matrix on a fuzzy soft multiset, and investigate how to use fuzzy soft matrices to solve decision making problems. An algorithm for a multiple choose selection problem is also provided. Finally, we demonstrate an illustrative example to show the decision making steps.

Mathematics Subject Classification 2010: 03E72, 15B15, 90C70, 62C86.

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Visualization of discrete random variables

Elena Gelova[†]
(joint work with Zoran Trifunov[†])

In this paper "Visualization of discrete random variables", by use of information technology and existing definitions, a discrete random variable will be introduced, with emphasis on variables modeling probability situations with only two outcomes, with probability p the event to occur and 1-p not to occur. The event can be repeated finite or infinite number of times. By analyses of examples, we will introduce Bernoulli, then binomial and we will finish with geometric random variable. We will enumerate discrete random variable with a geometric distribution, which can be visually displayed by an applet developed in GeoGebra that will make the visual representation of the problem, i.e. the areas of definition and the favorable events easier, then it will be resolved mathematically.

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Information visualization on the base of hierarchical graphs

Victor Kasyanov

Graphs are the most common abstract structure encountered in computer science and are widely used for abstract information representation [1], [2]. In the paper, we consider a practical and general graph formalism called hierarchical graphs. It is suited for visual processing and can be used in many areas where the strong structuring of information is needed. We present also the Higres and Visual Graph systems that are aimed at supporting of information visualization on the base hierarchical graph modes. Higres is a visualization tool and an editor for attributed hierarchical graphs and a platform for execution and animation of graph algorithms. Visual Graph was developed to visualize and explore large hierarchical graphs that present the internal structured information typically found in compilers.

Acknowledgements. The work was partially supported by the RFBR (12-07-0091).

Mathematics Subject Classification 2010: 68R10.

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Hacker attacks
Undetectable attacks from Trojans with reverse communication

Mane Piperevski

Computer integration in everyday human life create a motive for developing sophisticated and undetectable malicious codes, trojans with reverse communication that make use of deficiencies and vulnerability in the chain of security.

Mathematics Subject Classification 2010: 68P25, 94A60, 97P20.

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Mathematical Methods and Modelling in the Sciences

An effective algorithm for constructing (t, s)-sequences and (t, m, s)-nets over $\mathbb{Z}_b$

Vesna Dimitrievska Ristovska[†]
(joint work with Vasil Grozdanov[‡] and Antonio Atanasov[*])

In our talk we present an effective algorithm for construction of a special class of sequences and nets the so-called digitally (t, s)-sequences and (t, m, s)-nets in base b, where b is a prime. Our algorithm generalizes the construction of a LPτ-sequences, proposed by Sobol'. The proposed algorithm is based on using monocyclic differential operators over the field $\mathbb{Z}_b$.

For sufficient large dimension s and a small value of the quality parameter $t = 0$, $(0, s)$-sequences over the field $\mathbb{Z}_b$ exist, and can be constructed only for big enough base b. This is the main motivation of ours to consider this algorithm over the field $\mathbb{Z}_b$, where b is an arbitrary prime.

The proposed theoretical result is a sufficient condition so that the constructed sequence is (t, s)-sequence. The exactness of the parameter t is shown.

Two computer programs are written and presented. The first generates all primitive polynomials over the field $\mathbb{Z}_b$. The results of the first program are used in the process of work of the second program, which constructs (t, s)-sequences and (t, m, s)-nets in base b.

Mathematics Subject Classification 2010: 11K36,11K45,65C20

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Multi-criteria neutrosophic decision-making method based on accuracy function under neutrosophic environment

Rıdvan Şahin[†]
(joint work with Muhammed Yiğider[‡])

A neutrosophic set is a more general platform, which can be used to present uncertainty, imprecise, incomplete and inconsistent. In this paper an accuracy function for single valued neutrosophic sets is firstly proposed to make the distinction between them. Then the idea is extended to interval neutrosophic sets. A multi-criteria decision making method based on the developed accuracy functions is established in which criterion values for alternatives are single valued neutrosophic sets or interval neutrosophic sets. In decision making process, the neutrosophic weighted aggregation operators (arithmetic and geometric average operators) are adopted to aggregate the neutrosophic information related to each alternative. Thus, we can rank all alternatives and make the selection of the best of one(s) according to the accuracy functions. Finally, some illustrative examples are presented to verify the developed approach and to demonstrate its practicality and effectiveness.

Mathematics Subject Classification 2010: 06D72; 54A40

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The special role of the g-functions

Dhurata Valera[†]
(joint work with Ivi Dylgjeri[‡])

The class of g-functions by the g-generator of the system of pseudo-operations, apply a special role on functional equations and their solutions. More properties may be found in this class and by some elementary g-functions are given further studies to the entropy of $\oplus$-(decomposable) measure.

Mathematics Subject Classification 2010: 97170.

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Mathematical modeling of micropolar media and elements of thin-walled structures

Maria Varygina

Mathematical model of couple-stressed Cosserat continuum taking into account material microstructure is applied to describe the stress-strain state of composites, granulate, powdery and micropolar media. This model implicitly contains a small linear parameter – the particle size of the material. As a result to obtain correct numerical solution it is essential to perform computations on fine meshes using effective parallel algorithms for multiprocessor computers.

The parallel algorithm is based on the two-cyclic splitting method with respect to the spatial variables in combination with the explicit monotone finite-difference ENO-scheme. Computations of Lamb's problems on the action of instant concentrated load and the problem on the action of distributed periodic load of Π- and Λ-impulses on the surface of an elastic half-space are performed.

The proposed method can be applied in dynamic model of micropolar elasticity theory for orthotropic thin-walled structures. The numerical solution of wave propagation in elastic orthotropic rods with independent rotations and displacements is obtained.

The results of the analysis of the oscillation processes show that the considered media possess the eigenfrequency of acoustic resonance of rotational motion, which appears under certain conditions of perturbation and depends only on the parameters of the particles and the elasticity properties of the material.

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Mathematics Subject Classification 2010: 74B99, 74H15.

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Mathematical Finance, Actuarial Mathematics and Modelling in Economy

Compared forecasting of the macedonian central government debt during the period 2014-2018

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(joint work with Blagoja Spirkoski[‡] and Risto Malčeski[*])

The aim of this paper is to projection Central Government Debt of the Republic of Macedonia after the dual global financial and debt crisis. The object of the paper is to describe the challenges that have been faced by those working in public debt management during global and debt financial crisis. The authors of this paper explore the changes in Public and Central Government Debt management before, during to two crises and after the crisis with forecast for it from 2014 to 2018. For estimation and forecasting of public debt movements, decomposition of basic flows that lead to its change will be considered, and each one will be separately modeled. It is shown that in the period up to 2014 the share Central Government Debt in GDP in government scenarios does not exceed the margin of 40% (fiscal target), prescribed by Fiscal strategy of the Republic of Macedonia 2014-2016. In two-scenarios, by two different mathematical models, we get two-sided results, so in the first optimistic Gross Central Government Debt stays at the level of 35.0 to 40.0% of GDP as projection of IMF, while in second scenarios our forecasting with Dynamic Models it slightly exceeds the limit of 40% of GDP in analyses period 2014-2018.

Mathematics Subject Classification 2010: 91B84; 91G70.

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History and Education of Mathematics and Informatics

Methodical approach for the introduction and implementation of the exponential form of complex numbers in high school

Katerina Anevska

While studying complex numbers, high school students actually adopt only trigonometric and algebraic form of a complex number. Many non-standard geometry problems can be solved using complex numbers, where the exponential form and Euler complex formulas have an important role. However, they are not taught in high school. In this paper we provide a methodological approach that would justify the introduction of an exponential form of a complex number and Euler formulas and their application in the study of Euclid geometry using complex numbers.

Mathematics Subject Classification 2010: 97D40, 97I80.

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One binary lesson

Rumjana Angelova

The article discusses one binary lesson of mathematics and physics with students from 10th grade of Vocational School of Economics and Management Pazardzhik, Bulgaria, it presents a preparation, describes the implementation, considers the difficulties, analyzes the results. Mathematical ideas and structures and the problem of radioactive decay are considered in synthesis.

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Formalism in knowledge of mathematics

Tatjana Atanasova-Pačemska[†]
(joint work with Limonka Lazarova[†], Zoran Trifunov[†] and
Marija Miteva[†])

Information obtained in our work with the students over a long period indicate that the students have distorted and incorrect knowledge of basic mathematical terms and concepts. In order to determine the situation, the diagnostic test has been made for the students in the first year of studies at the Faculty of Economics and the Faculty of Computer Science. The diagnostic test is consists of questions from algebra, geometry and real analysis. We need the information from this test to determine the flaws in the mathematical formalism and knowledge. The diagnostic test will be carried out electronically (the patterns of external testing). The results will be analyzed and presented after which conclusions and recommendations will be given.

Mathematics Subject Classifcation 2010: 97D70.

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Developing communication competencies in natural and foreign language in the context of mathematics education

Toni Chehlarova[†]
(joint work with Evgenia Sendova[†])

The paper presents the experience of the authors related to the development of key competences in the STEAM education in the context of the European projects KeyCoMath [1] and KeyCoNet, as well as within two gifted education programs in Bulgaria and USA. Resources of the Virtual Mathematics Laboratory (VirMathLab) [2] http://www.math.bas.bg/omi/cabinet/ will be discussed with emphasis on enhancing the communication competences in natural and foreign language.

Mathematics Subject Classifcation 2010: 97U30

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Interdisciplinary (mathematics, physics and biology) national testing in Romania – personalized teaching and learning plans

Bogdan Cristescu[†]
(joint work with Gabriela Streinu-Cercel[†], Liviu Blanariu[‡] and Daniela Blanariu[*])

We present a national testing strategy in Romania, designed to measure what the learners of the 6th degree (12-13 years old) are able to do with the content of mathematics, physics and biology already learnt and to what degree the learners connect those contents.

With the increasing importance given to standardized international and national testing, in 2014, we implemented in Romania a national testing program using a unique test combining elements from mathematics, physics and biology, for 12-13 year-old students. We present the stages of implementing this testing. We defined six interdisciplinary competencies [1] and we used a three-parameter logistic model [2] to describe the link between the students' performance in tests and their corresponding abilities in mathematics, physics and biology. The novelty of this strategy resides in the manner of interpreting test results. Instead of being graded, each test is evaluated based on clustered codes, providing individual feedback regarding each student (e.g., the curricular area where further work is needed - personalized learning plans) as well as clustered feedback at class, school, regional and national level, identifying the areas where adjustment would increase the effectiveness of the teaching process - personalized teaching plans. To our knowledge, this is the first nationally-applied strategy for personalizing the teaching and learning process.

Mathematics Subject Classifcation 2010: 97D60, 97B20

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Intellectual and emotional maturity of pupils in solving time-measuring problems

Olivera Đorđević

Primary school pupils meet the notion of the measuring the time in the second grade. In this paper we investigate their capabilities to understand and solve time-measuring problems. We analyze abstract situations, use didactic tools, and correlate the program of mathematics with other subjects.

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Creation of the contextual tasks bank with economic content for the formation of the financial literacy of students

Larisa Forkunova

At the moment the great attention in Russia and other countries is paid to the low level of financial literacy problem of the population. One of the objectives of financial education is to teach children to use the mathematical methods for the economic decisions. As one of the means to achieve this purpose we use contextual math tasks. Our purpose was to create the contextual math task bank with economic content and database for it. For further applications in life of the received skills, the students, working with contextual task, must learn to select data themselves, from a plurality of available, important from their point of view. Our purpose was to test tasks bank and database which we have developed. In several testing groups of different ages, we studied how children are ready to make a conscious choice of the necessary data for decision-making. One more of purposes of testing was to identify what data is important and what is not taken into account in dealing by the students of a particular contextual task.

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The integration of mathematics instruction in elementary education

Metodi Glavče[†]
(joint work with Katerina Anevska[‡] and Risto Malčeski[‡])

The division of mathematics in several scientific disciplines in the previous century resulted in significant differentiation in the mathematics education in primary and high school education, something that can be noticed in elementary education as well. The differentiation comes with many shortcomings, which primarily reflect in the insufficient development of the cognitive properties, and in the inability to present a complete and uniform illustration of nature to the young generations. Hence, there is a need to improve the integration of instruction, not only of the inter-subject and intra-subject, but also of the integration of the instruction with the immediate environment. This is very important in terms of the elementary education. In this paper we will analyze the integration of mathematics instruction, and we will present examples, which may serve as guidelines for its successful realization.

Mathematics Subject Classification 2010: 97B70, 97C70.

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Centers of homothety of circle configuartions

Sava Grozdev[†]
(joint work with Veselin Nenkov[‡])

Each triplet of circles in the plane defines six points, which are the homothetic centers of the circle pairs. The paper examines geometric properties of the 6-point location. Further results are proposed for more than three circles.

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Logical tasks – real challenge for the youngest scholars

Valentina Gogovska

School mathematics, due to its specificity, possesses great opportunities for scholar's intellectual development which can be fully accomplished through prior organization of the educational process. From this point of view, the conclusion of Vygotsky - Leontev's school of psychology according to which the child's development occurs in a process of adopting historically created mathematical knowledge, skills and habits is extremely important. Development is the result of the active work performed by a child who has mastered the abilities for orientation and training, which then gradually turn into a form of their own independence. In this instance, the basis of developmental teaching is its content. The aim of this paper is introducing logical tasks for elementary scholars like one possible solution for conducted research at the beginning of the school year 2011/12 in several schools in R. Macedonia, based on a previous research conducted in The Netherlands within the MORE project. Because of that situation [1] two questionnaires (for teachers and scholars) were made and some logical tasks are shown. In this paper I can conclude that logical tasks have to be an option. Nowadays first graders (youngest scholars) has Kangaroo, but they need much options, some mathematics newspaper, logical computer games, homework's, etc.

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Constructive geometry and technical drawing

Jitka Hodaňová

Greek mathematicians studied plane geometry, solid geometry, spherical trigonometry, optics and we can also find their perspective origins and they studied fields and curves. The geometry development influenced other disciplines − astronomy, physics and other technical disciplines. The technical practice like building required to display three-dimensional objects in that way, that builders could read the shape and dimension of an object from drawings (see well-kept historical monuments). The plane curve studies have their origin in ancient world. Their names like Diokles' kissoide, Archimedes' spiral, Perseus' curve show, that mathematicians developed the curves in ancient world.

A curve can be defined as a set of points on a plane or in space which originates as a trajectory of a moving point. Real objects, modelled upon geometric curves, have only one dominant dimension, the others being insignificant. The constructional law, according to which a curve generates, can be described either synthetically, or analytically, using point and vector functions. The submitted article deals with the kinetics of objects, special attention being paid to the movement on a plane.

Mathematics Subject Classification 2010: 97G80.

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Teaching mathematics online - new practice in high school

Teuta Iljazi

New technology revolutionizes teaching and learning process. The gap between curriculum in the school and situation in society, learners' 21st century skills development, learning opportunities maximization are facts behind the use of new technology in Mathematics teaching and learning. The goal of this study is to prove the hypothesis that asynchronous online mathematics teaching improves the success in Mathematics. Technology in teaching and in learning process provides benefits as connectivity, flexibility, interactivity, motivation, collaboration which results on better knowledge, higher success in Mathematics and new experience for students and new tools for learning.

The sample of the study consist of third classes high school students who actively participate in online asynchronous learning process. Teaching Mathematics online is practiced twice per week as consultation hours during second semester of third year. Moreover I have the opinion of students who are part of this study about this kind of teaching tool. They want to use this kind of tool in every subject, since they benefit from it in different ways.

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The nature of mathematics and mathematics learning: Prospective primary school teachers' and pre-school teachers' beliefs at the beginning of initial teacher education

Slagjana Jakimovik

Mathematics education of primary school teachers and pre-school teachers is of a great importance for teachers' preparedness to support the development of mathematical thinking of young learners. Although mathematics content knowledge is a major component of the professional body of knowledge required for teaching mathematics, teachers' professional beliefs on what is mathematics and how mathematics is learned have a significant mediating effect on teachers' success in providing genuine opportunities to learn meaningful mathematics.

Analysis of prospective teachers' beliefs on the nature of mathematics and on mathematics learning when they enter initial teacher education was the research goal of the study. It was conducted at the beginning of the second semester, when students encounter the first mathematics course for teachers. The student questionnaire consisted of parts of the questionnaire used in the international study TEDS-M and of a small number of mathematics items designed to verify the answers given by the questionnaire respondents. The results revealed a difference between the self-professed beliefs of the students and the approaches they used to respond to the mathematics items. These findings point to the need for making provisions within initial teacher education to help future teachers in developing mathematical knowledge for teaching and consistent professional beliefs.

Mathematics Subject Classification 2010: 97 Mathematics education.

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Application of the package *Mathematica* for examining features and sketching the graph of a real function of a real variable

Sanja Kostadinova Atanasova†
(joint work with Katerina Hadži-Velkova Saneva† and
Sonja Gegovska-Zajkova†)

In this paper we describe how the package *Mathematica* can be used as an auxiliary tool in teaching mathematics in fourth year high school. From the topics that are taught there, we show on specific examples how the package *Mathematica* can be helpful when examining the properties and sketching the graph of a real function of a real variable. The results are made in *Wolfram Mathematica 8*.

Mathematics Subject Classification 2010: 97M99, 97Q60.

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A study on the concept of function

Limonka Lazarova[†]
(joint work with Tatjana Atanasova-Pačemska[†], Marija Miteva[†] and Zoran Trifunov[†])

The concept of function plays an important role all through the mathematics curriculum. It is vital for the students' ability to describe relationships of change between variables, explain parameter changes and interpret and analyze graphs. However, it is a confusing concept for the students. Therefore in this paper we consider the concept of function and the way of study throughout history as well as today. In order to facilitate a study of this concept, we give suggestions and recommendations to the teachers.

Mathematics Subject Classification 2010: 97D70, 97D80.

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Application of e-technologies in formative assessment

Jasmina Markoska[†]
(joint work with Gjorgji Markoski[‡])

The objective of formative assessment is a learning process and that provides feedback about learning itself, both for the student and the teacher. In this paper we present a way to deliver formative assessment using interactive quizzes.

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Posing problems in the form of games

Jasmina Milinković

Problem posing has been identified as important aspect of education for mathematics teachers. The paper explores the possibilities of designing authentic mathematical problems in the form of game. The starting point of this paper is theoretical approach to posing problems (Kilpatrick, Silver). The next part illustrates the approach to designing problems based on the transformation of mathematical problems into a form of game. The games presented in the paper may be used in the teaching of mathematics with the objective of practicing math skills and of developing mathematical logical reasoning. Finally, we exemplify the process of creation of games/problems via transformation. After taking into consideration a basic game, we point to different variants of the game created by transformation of the original one. The aim of the paper is to encourage researchers and practitioners to explore the possibilities of enriching the teaching of mathematics problems in the form of games.

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Stealing and abusing mathematical results

Marija Miteva[†]
(joint work with Limonka Lazarova[†], Tatjana Atanasova-Pačemska[†] and Zoran Trifunov[†])

During the history of mathematics there exist numerous examples of stealing mathematical formulas and theories as well as abusing mathematical results. In this paper we give some examples of stealing mathematical results, including famous scientists with famous mathematical achievements. We give also examples for abusing mathematics and its results.

Mathematics Subject Classification 2010: 01A99.

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The role of the teacher in working with talented students

Cana Naumovska[†]
(joint work with Dančе Sivakova[‡] and Jovanka Vangelova[*])

In the educational system of a country, the work with talented students is considered to be a very important component. The stimulation of the gifted students is the best investment in the society, since the fruits of this engagement contribute to its progress. The question is what is the treatment of talented and gifted students in our schools. Observations of the characteristics of the educational system, the situation and the conditions in which gifted students study in our schools, indicate that the system of measures aimed at stimulating their development is not operationalized enough. Many things are not in favor of gifted students, including: classes with too many students, uniform curricula, underdeveloped individual work, inability to select the appropriate teacher(s), etc.

Only the declarative support for the gifted students is present in our schools. "Brain drain" is just one indication of the problem with our talented students. Worldwide, even in our immediate environment, this issue has already been considered. The issue of establishing an adequate treatment of gifted students should be out of the margins and set the early agenda of social interest. In our country it is necessary to develop a national strategy that will operationalize the treatment of the gifted students, and to use all the social, institutional and other mechanisms that will keep them in our country.

Mathematics Subject Classification 2010: 97B99.

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Interactive graphics

Biljana Načevska[†]
(joint work with Sonja Gegovska-Zajkova[†])

One of the the most common application of Mathematica in educational purposes is crating interactive graphics. Here, using the Classroom Assistant and the commands Manipulate and Animate starting from the graph of a single real variable function, we obtain different types of interactive graphics. Thus, students can be able to visualize the impact of different parameters on the curve shape.

Mathematics Subject Classification 2010: 97U50, 97U70.

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Wolfram Mathematica's CDF-format and how to use it in the classroom

Biljana Načevska[†]
(joint work with Sonja Gegovska-Zajkova[†])

Wolfram Mathematica's CDF format of a document is a very easy and extremely usable way of presentations and preparing lectures for the students. PDF and Power Point Presentation can only show static images, which is very textbook-like way of presentation. On the other side, CDF enables introducing dynamics in presentations. Moreover, it provides interactive data view, which is much more effective way of learning. It is possible to create unique e-Book, presentation, lecture notes, tests, journal articles and reports, all in one, with CDF. We are going to show how it can be used by students in order to investigate and construct their own knowledge.

Mathematics Subject Classification 2010: 997U50, 97U70.

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Why they need proofs in mathematics teaching?

Aslanbek Naziev

The question posed in the tittle is one of the most frequent in the discussions about mathematics teaching. In fact, why the proofs? Why not let the students to believe the teacher? Try to understand this. Look at the following example. Can one believe that a straight line and a circle can have only one common point? No! We can draw thousand times a line and a circle and in every case we will see that the line and the circle either have no common points or have precisely two common points or have the whole segment of common points. Well, we cannot believe this but we can be convinced of that and understand that by discovering that — with the help of the proof.

Now, we can indicate the three high aspirations of the proof in mathematics and mathematics teaching: to lead to discovery, to detach the discovery and to help to understand the discovery.

In the full paper I intend to examine the question in much more detail.

Mathematics Subject Classification 2010: Primary 97D20; Secondary 97E50.

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Direct and indirect methods of proof. The Lehmus-Steiner Theorem

Julia Ninova[†]
(joint work with Vesselka Mihova[†])

In the paper there are discussed the notions "direct" and "indirect" proof of a given statement. A direct geometric proof of the Lehmus-Steiner Theorem is proposed.

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GEOTHNK - semantic pathways for building a spatially-thinking society

Nataliya Pavlova[†]
(joint work with Dragomir Marchev[‡])

The basic concepts and ideas of a new European project called GEOTHNK are presented in the paper. It focuses on education in natural science and mathematics with students of all ages.

The main aim of this project is to enhance spatial thinking through an innovative ICT-based approach and an open, collaborative educational environment. The idea is to offer a methodological approach which allows the interdisciplinary organization and semantic linkage of knowledge.

We will emphasize the following objectives:

- Development of a pedagogical framework based on spatial thinking.
- Design of learning pathways based on the proposed framework.
- Development of the GEOTHNK Platform.
- Formulation of a semantic network to provide a dynamic structure facilitating knowledge visualization and exploration.
- Development of a systematic evaluation and validation approach of the proposed activities in order to identify their impact.

The consortium of the project consists of eight organizations from six counties. The official site of the project is http://www.geothnk.eu/.

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Application of Web 2.0 in tuition of math and IT technology

Nadica Piperevska Cvetkovska[†]
(joint work with Danijela Đorđević[†])

With the project of the Ministry of Education and Science "Computer for Every Child" new project "Tablet for Every Child", growing number of schools in Macedonia are equipped with modern computers that are connected to the Internet, and teachers and students are trained to use computers in teaching.

As a results of this we want to share some Web 2.0 teaching tools that are some of the best-of-the-best ones that have worked for us, our students, and other teachers. Web 2.0 can provide compelling teaching and learning opportunities. These quality and affordable applications to help teachers easily explain concepts, while students successfully learn the material.Theachers can revitalize their lessons by integrating these instructional technology tools into their teaching.

Mathematics Subject Classification 2010: 97P50, 97Q20, 97R20.

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For the paradox of Cardano

Boro M. Piperevski

In this paper, Cardano formulas are processed for the roots of third degree algebraic equation in dependence on discriminant. By that the notion of algebraic solutions to third degree equations in radicals is discussed, in correlation with the paradox of Cardano. This fact is not explicitly noted in literature.

Mathematics Subject Classification 2010: 11D25, 97U70.

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An approach to elimination of «experimental-theoretical ruptur» in teaching mathematics with using DGS and evaluation of its effectiveness

Maria Shabanova[†]
(joint work with Alexander Yastrebov[‡] and Tatiana Shirikova[†])

From the 80ths of XX century dynamical geometry software (DGS) are used in mathematics education. DGS allow us to create dynamic drawings and experiment with them to obtain and verify hypotheses concerning the properties of mathematical objects. Easy experimental way of knowing, a high level of credibility to dynamical visualization lead to the fact that the view that deductive reasoning is no longer needed spread. This phenomenon was named "experimental-theoretical rupture" in the scientific literature. This name emphasize that one of the dualistic properties of mathematics is violated. The article presents the results of experimental and theoretical research. The purpose is to find a pedagogical technology of using DGS in the composing of mathematical propositions that eliminates this risk. Authors' technology consists of three steps. The first step: students learn how to make right conclusions from computer experiments and how to take into consideration the limited possibilities of these experiments to verification of the propositions. The second step: students learn how to use theoretical methods to verify the correctness of the experiment and the adequacy of the dynamical drawings. The third step: students learn how to use a computer experiment to find ideas for using theoretical methods in proving propositions.

Mathematics Subject Classification 2010: 97G40, 97R20.

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The teacher as facilitator in use of IT in teaching mathematics

Zoran Trifunov[†]
(joint work with Tatjana Atanasova-Pačemska[†], Limonka Lazarova[†] and Marija Miteva[†])

In educational process, the teachers should use informatics technology. The use of the technology provides new form of teaching with emphasis on the student, in which with innovative use of ICT by every student the traditional way of learning is bettering, which isnâĂŹt expressed only by use of LCD projectors and presentations from the teacher, at which the students are passive and donâĂŹt take part in mastering the education material. Informatics technology, especially the computers that are in our classrooms, helped for active inclusion of the students in mastering the material, through making some applets on the softwares that are installed on the computers or to use them for searching on the internet about the teaching materials. With that, they actively take part in mastering the new teaching contents, and the teacher as facilitator, just gives guidelines for the making of the applets, controls found contents on the internet, and advices and encourages the students for using the computers. This way the teacher acquires alternative ways for getting the solutions and full support on the students in using the computers. In this paper we will see how the teacher as facilitator, will introduce the students to rectangular coordinate system and quadrants and also to visually determining the origin point in any quadrant.

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IV Congress Workshop

Getting Started with GeoGebra

Workshop goals. To introduce the users with the meaning of dynamic software for mathematics and its value in learning mathematics. To demonstrate the easy and intuitive GeoGebra interface and how GeoGebra relates to algebra, geometry, measurement, functions and tables, and to further mathematical understanding and logical thinking skills.

Motivation. A modern teacher needs to be allowed to use information technologies and free educational softwares for successful realization of the classes. GeoGebra is one of the free softwares that can solve and visually show mathematical problems. Some workshops, intended for a basic and advanced level of use on GeoGebra, develop applets that concerns problems from geometry, algebra and calculus.

A brief description of the workshop. This workshop is intended for absolute beginners and for users of GeoGebra that wants their students to learn how to use this software package as well. It covers the basics of GeoGebra in a structured, but completely interactive format with 6 threads, i.e. the participants will learn how to use this software package by creating 6 GeoGebra worksheets. A power point presentations are integrated in the workshop with a corresponding handouts. Reference materials and worksheets will be included with the registration materials.

A brief description of GeoGebra. GeoGebra is a dynamic software for mathematics that can be easily implemented in math classes. It joins geometry, algebra and calculus. On one hand, GeoGebra is an interactive geometry system. You can do constructions with points, vectors, segments, lines, conic sections as well as functions while changing them dynamically afterwards. On the other hand, equations and coordinates can be entered directly. Thus, GeoGebra has the ability to deal with variables for numbers, vectors, and points. It finds derivatives and integrals of functions and offers commands like Root or Vertex. These two views are characteristics of GeoGebra: an expression in the Algebra View corresponds to an object in the Graphics View and vice versa.

A brief description of the organizer's background. Zoran Trifunov, zorantrifunov@gmail.com, http://geogebramkd.wikispaces.com/

Workshop requirements: basic computer skills, no pre-knowledge of dynamic math applications or pre-knowledge of GeoGebra. Participants should carry their own laptop.

Lecturers:

Zoran Trifunov, Goce Delčev University, Štip
Linda Stojanovska, St. Kliment Ohridski University, Bitola
Elena Gelova, Goce Delčev University, Štip

Workshop schedule:

1. Introduction. Starting and setting GeoGebra
2. Point in quadrant
3. Examining polynomials
4. Square function, equation and non-equation
5. Random variables
6. Parametric given curve

Following this course, participants will:

- Develop familiarity with available GeoGebra Views, geometric tools, as well as the use of basic commands.
- Develop the ability to change the properties (colors, shading, thickness, etc.) of objects, in order to create appealing instructional materials.
- Use GeoGebra to create files, export static images and insert them into a word processing document, as well as to create interactive web pages by uploading your files to GeoGebraTube.
- Develop familiarity with the use of algebraic expressions (using GeoGebra like a graphing calculator) in the Input Bar.
- Develop familiarity with the use of commands in the Input Bar.
- Develop familiarity with the use of advanced features like animations, sequences, dynamic text, and conditional visibility.
- Develop the ability to create User Defined Tools and ability to customize the user interface of GeoGebra (e.g. Tool Bar).
- Develop familiarity with the features of GeoGebraTube.
- Create a collection of instructional materials on GeoGebraTube.

Participants will demonstrate an understanding of how to use GeoGebra for math classes teaching and learning by:

- Learning about the dynamic capabilities of GeoGebra, especially of the potential of interconnected multiple and dynamic representations for mathematics teaching and learning.

- Learning how to use GeoGebra as a demonstration and presentation tool with existing (ex. GeoGebraTube) and self-made instructional materials.
- Learning how to select and use of existing GeoGebra materials (ex. GeoGebraTube).
- Building confidence in the ability of using GeoGebra and to let students work with prepared GeoGebra materials.
- Building confidence in the ability to introduce students to GeoGebra and guiding them towards an independent use of GeoGebra while supporting learning by discoveries and mathematical experiments.

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