An algebraic representation of a class of homogeneous Steiner quadruple systems

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A Steiner system $S(t,k,v)$ is a pair $(Q,\mathcal{B})$, where Q is a v-element set and $\mathcal{B}$ is a collection of its k-element subsets (called blocks), such that every t-element subset of Q is contained in exactly one block. Steiner systems $S(2,3,v)$ are known as Steiner triple systems (briefly STS) and their algebraic representatives are the idempotent totally symmetric quasigroups. Steiner systems $S(3,4,v)$ are called Steiner quadruple systems (SQS). Algebraic structures which correspond to them are the idempotent totally symmetric ternary quasigroups.

For a given Steiner quadruple system $(Q,\mathcal{B})$ and $a \in Q$, by taking the set $Q \setminus \{a\}$ and the blocks $\{\{x,y,z\}\{x,y,z,a\} \in \mathcal{B}\}$, a Steiner triple system is obtained. It is called a derived triple system (DTS) of the quadruple system $(Q,\mathcal{B})$. An SQS is called homogeneous if all of its derived triple systems are isomorphic. If all derived triple systems of an SQS are pairwise nonisomorphic, then the quadruple system is called heterogenous.

In this paper sufficient conditions for SQS to be homogenous are given, resulting with an algebraic representation of one class of homogenous quadruple systems.