Vector valued hyperstructures

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Basic concepts of vector valued hyperstructures, i.e. (n,m)-hyperstructures, are introduced. Namely, let $[\ ]$ be a mapping $[\ ] : H^n \to (\mathcal{P}^*(H))^m$ from the n-th cartesian product of H to the m-th cartesian product of $\mathcal{P}^*(H)$, where $\mathcal{P}^*(H)$ is the set of all nonempty subsets of H. Then $[\ ]$ is called an (n,m)-*hyperoperation* on H or, if it is not necessary to emphasize the integers n and m, then we will say that $[\ ]$ is a *vector valued hyperoperation* instead of (n,m)-hyperoperation. We can associate to the operation $[\ ]$ a sequence of m n-ary hyperoperations $[\ ]_s : H^n \to \mathcal{P}^*(H)$, $s \in \{1,2,\ldots,m\}$, by putting

$$[a_1 \ldots a_n]_s = B_s \quad \Leftrightarrow \quad [a_1 \ldots a_n] = (B_1, \ldots, B_m),$$

for all $a_1,\ldots,a_n \in H$. Then, we call $[\ ]_s$ the *s-th component hyperoperation* of $[\ ]$ and write $[\ ] = ([\ ]_1, \ldots, [\ ]_m)$.

An algebraic structure $\boldsymbol{H} = (H, [\ ])$, where $[\ ]$ is an (n,m)-ary hyperoperation defined on a nonempty set H, is called an (n,m)-*hypergroupoid* or *vector valued hypergroupoid*.

We define (i,j)-associative and weekly (i,j)-associative (n,m)-hyperoperation, (n,m)-hypersemigroup and week (n,m)-hypersemigroup, (n,m)-hypergroup and week (n,m)-hypergroup, cancellative, partially cancellative and strongly cancellative (n,m)-hypergroupiod, weak neutral element and neutral element. Some of their properties are investigated for $n = m + k$, $k \geq 1$ and many examples are given.