On the concept of connectedness

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The definition of connectedness is given in the beginning of 20th century by Riesz and Hausdorff.

The definition given earlier by Cantor in the period from 1879 until 1884 is:

A space is connected if for any two points x and y and any $r > 0$ there is a finite number of points $x = x_0, x_1, x_2 \ldots, x_n = y$ such that $d(x_i, x_{i+1}) < r$.

The two definitions coincide in the case of compact metric spaces.

Here are reformulations of Cantor definition of connectedness:

1) A space is *connected* if for any two points x and y and any $r > 0$ there is a finite chain of r-balls from x to y.

2) A space is *connected* if for any two points x and y and any open covering of X there is a finite chain of members of the covering from x to y.

The reformulation 2) coincides with the usual Riesz-Hausdorff definition for all topological spaces. This definition has advantages in some situations, for example a simpler definition of qasicomponents.

Theory of shape was introduced by K. Borsuk at the end of sixties of 20th century, as a better tool than homotopy theory for study of spaces with bad local properties. In that time the notion of *pointed 1-movability* (*strong connectedness*) was introduced also by Borsuk. The original definition uses embeddings of compact metric spaces in Hilbert cube.

Intrinsic definition of shape uses only coverings of a space and this is the reason why definition of connectedness only by coverings is the appropriate one and corresponds to intrinsic approach.

In the light of intrinsic approach the notion of pointed 1-movability will be reformulated by use only of coverings of the space and will be explained why the notion deserves name strong connectedness. It will be compared the notion of path connectedness in homotopy theory, the notion of connectedness in shape theory and the notion of strong connectedness (pointed 1-movability) in strong shape theory.