Structure theorems for G-type spaces of ultradistributions over $\mathbb{R}^d_+$

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Two structure theorems for the G-type spaces of ultradistributions over $[0,\infty)^d$, i.e. $(G^\alpha_\alpha(\mathbb{R}^d_+))'$, $\alpha \geq 1$ are given. The first one states that $f \in (G^\alpha_\alpha(\mathbb{R}^d_+))'$, $\alpha \geq 1$ if and only if it can be written as

$$f = \left(\sum_{k\in\mathbb{N}^d_0} c_k \left(xD^2 + D - \frac{x}{4} + \frac{1}{2} \right)^k \right) F,$$

where the coefficients c_k have a suitable growth and $F \in L^2(\mathbb{R}^d_+)$. The second one uses the fact that $G^\alpha_\alpha(\mathbb{R}^d_+)$, $\alpha \geq 1$ is given as an injective inductive limit of Fréchet spaces and loosely speaking represents $f \in (G^\alpha_\alpha(\mathbb{R}^d_+))'$, $\alpha \geq 1$ by giving its action on each layer of the inductive limit in the following way

$$\begin{aligned} \langle f, \phi \rangle &= \sum_{p,k\in\mathbb{N}^d_0} \int_{\mathbb{R}^d_+} F_{A,p,k}(x) x^{(p+k)/2} D^p \phi(x) dx \\ &\quad + \sum_{|m|\leq j, |n|\leq j} \int_{\mathbb{R}^d_+} \tilde{F}_{A,n,m}(x) x^m D^n \phi(x) dx, \end{aligned}$$

where the $L^2(\mathbb{R}^d_+)$-functions $F_{A,p,k}$ and $\tilde{F}_{A,n,m}$ depend on the layer.