On general Stieltjes moment problems

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The problem of moments, as its generalizations, is an important mathematical problem which has attracted much attention for more than a century. It was first raised and solved by Stieltjes for positive measures. Boas and Pólya, independently, showed later that given an arbitrary sequence $\{a_n\}_{n=0}^{\infty}$ there is always a function of bounded variation F such that

$$a_n = \int_0^\infty x^n \mathrm{d}F(x)\,, \quad n \in \mathbb{N}\,. \tag{1}$$

A major improvement to this result was achieved by Durán, who was able to show the existence of regular solutions to (1). He proved that in fact every Stieltjes moment problem

$$a_n = \int_0^\infty x^n \phi(x)\mathrm{d}x\,, \quad n \in \mathbb{N}\,, \tag{2}$$

admits a solution $\phi \in \mathcal{S}(0,\infty)$, that is, a solution in the Schwartz class of rapidly decreasing smooth functions with $\operatorname{supp}\phi \subseteq [0,\infty)$.

In this talk we discuss an approach to abstract moment problems that leads to "if and only if" criteria for solvability. In particular, we shall replace the sequence of monomials in (2) by a rather general sequence of distributions $\{f_n\}_{n=0}^{\infty}$ with $\operatorname{supp} f_n \subseteq [0,\infty)$ and present a complete characterization of those $\{f_n\}_{n=0}^{\infty}$ such that every generalized Stieltjes moment problem

$$a_n = \langle f_n, \phi\rangle\,, \quad n \in \mathbb{N}\,,$$

has a solution $\phi \in \mathcal{S}(0,\infty)$. The talk is based on collaborative works with R. Estrada, and A. Debrouwere.