Union of Mathematicians of Macedonia
VI Congress of Mathematicians of Macedonia

BOOK OF ABSTRACTS

June 15-18, 2016, Ohrid, Republic of Macedonia

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EDITORIAL COMMENT Book of Abstracts CMM 2016

We are proud to present the Book of Abstracts of the VI Congress of Mathematicians of Macedonia organized by the Union of Mathematicians of Macedonia. Since 1996 there have been five congress meetings that were held in Ohrid (1996 and 2000), Struga (2005 and 2008) and Ohrid (2014). The VI Congress has an international character and it is open to all areas of mathematics and its applications. It is a great opportunity to meet other colleagues, to share experiences and foster collaboration.

The VI Congress agenda will include plenary lectures, short communications and poster sessions, organized in the following sections:

- 1. Algebra, Discrete Mathematics, Combinatorics and Number Theory;
- 2. Complex Analysis, Real Analysis and Functional Analysis;
- 3. Differential Equations and Dynamical Systems;
- 4. Topology, Geometry, Differential Geometry and Mathematical Physics;
- 5. Probability, Statistics, Numerical Mathematics and Optimization;
- 6. Actuarial Mathematics, Mathematical Finance and Modeling in Economy;
- 7. Mathematical Aspects of Computer Sciences;
- 8. History of Mathematics and Mathematics Education,

as well as two international workshops:

- International Workshop on Generalized Functions and Pseudo-differential Operators
- 2. Fixed point theory and its applications

and two seminars for teachers:

- 1. Enhancing mathematical education through combinatorics.
- 2. The role of elementary number theory in teaching mathematically gifted students: opportunities and challenges.

This publication gathers all the abstracts that are accepted for presentation at the VI Congress. They are abstracts of 6 plenary lectures, 85 short communications and poster presentations on recent results and future challenges and 29 communications from 2 international workshops. All the abstracts are listed in alphabetical order in each section, including the plenary lectures and workshop communications.

This Book of Abstracts builds the basis for the proceedings to follow. All authors are invited to submit full papers that will be published after a peer review process in:

- Matematički Bilten,
- Special issue of Matematički Bilten devoted to Section 8
- Proceedings of the VI Congress.

The Editor

PLENARY LECTURES

(in alphabetical order)

Dončo Dimovski

Codimension one coincidence indices

Smile Markovski

Applications of quasigroup string transformations – results of 20 years of investigation of Macedonian researchers

Veselin Nenkov

Invariant theorems in Euclidean geometry with respect to conics

Stevan Pilipović

G -type spaces of utradistributions over \mathbb{R}^d_+

Luigi Rodino

Some remarks on the recent history of the partial differential equations

Nikita Shekutkovski

On the concept of connectedness

Codimension one coincidence indices

Dončo Dimovski

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Using the geometric description of spin manifolds and spin structures, a generalization of one-parameter fixed point indices is obtained for codimension one coincidences.

Let $F, G: X \to Y$ be PL maps where X and Y are closed, connected, spin PL manifolds, X is (n+1)-dimensional and Y is n-dimensional, and $n \ge 4$. A coincidence of F and G is a point $a \in X$ such that F(a) = G(a). The set of all the coincidences is denoted by Coin(F,G). For a family V of isolated circles of coincidences of F and G, two indices are defined: $ind_1(F,G;V)$ - which is an element in the first homology group $H_1(E)$, where E is the space of paths in $X \times Y$ from the graph of F to the graph of G; and $ind_2(F,G;V)$ - which is an element in the group 9_2 with two elements.

Theorem. For a family V of isolated circles of coincidences of F and G in the same coincidence class there is a neighborhood N of V and a homotopy from F to H rel $X \setminus N$ such that $Coin(H,G) = Coin(F,G) \setminus V$ if and only if $ind_1(F,G;V) = 0$ and $ind_2(F,G;V) = 0$.

Applications of quasigroup string transformations

- Results of 20 years of investigation of Macedonian researchers

SMILE MARKOVSKI

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Many applications of mathematics use transformations of strings of symbols. Given a nonempty finite set $A = \{a_1, a_2, ..., a_n\}$ of symbols (or letters) a_i , denote by $A^n = \{(a_1, a_2, ..., a_n) | a_i \in A, i = 1, 2, ..., n\}$ the n-th power set of A. We usually identify the n-tuple $(a_1, a_2, ..., a_n)$ with the string $a_1a_2...a_n$ and we denote by $A^+ = \bigcup_{n>0} A^n$ the set of all nonempty strings over the set A. Any mapping $f: B \to A^+$, where $B \subseteq A^+$, is said to be a transformation of strings of symbols of A.

We mention here two important fields where applications of transformations of strings of symbols are essential. One is the cryptography and the other is the coding theory.

There are several cryptographic products or crypto-primitives based on such transformations. As simple examples, let mention that a cipher is a mapping $f: A^+ \to A^+$ such that the original message $a_1a_2 \dots a_n$ cannot be effectively recovered by knowing only the cipher-text $f(a_1a_2 \dots a_n)$, and a hash function $f: A^+ \to A^k$ has, among others, the property to be effectively impossible to find two messages m_1 , $m_2 \in A^+$ such that $h(m_1) = h(m_2)$, i.e., with same message digest.

In coding theory one has to recover the message m sent through an insecure channel from the message m' obtained at the end of the channel after transmission. Here, at first the original message $m=m_1m_2\dots m_k$, $m_i\in A$, is expanded to a codeword $c(m)=c_1c_2\dots c_n,\ c_i\in A$, where k< n. Then the codeword is sent through an insecure channel and a message $m'=d_1d_2\dots d_n$ is obtained, usually different than c(m), since the channel produces errors $(d_i\neq c_i)$ for some indexes i). Now, the coding $c:A^+\to A^+$ has to be defined in such a way the probability of recovering the original message m from the obtained m' to be as high as possible.

Twenty years ago a group of mathematicians from the Institute of Informatics of the Faculty of Natural Sciences and Mathematics at University in Skopje started to investigate transformations of strings by using quasigroups. Given a quasigroup (Q,*), several quasigroup string

transformations $f: Q^+ \to Q^+$ can be defined, and it happened the so called e-transformations and d-transformations to be most useful. For example, an e-transformation is defined by $e(a_1a_2...a_n) = b_1b_2...b_n$, $a_i, b_i \in Q$, if and only if $b_j = b_{j-1} * a_j$, j = 1, 2, ..., n and b_0 is some fixed element of Q. Then $e: Q^+ \to Q^+$ is a permutation of Q^+ that has several useful properties, suitable for applications in cryptography and coding theory. Based on quasigroup string transformations there were designed many different types of crypto-primitives and new error detecting and error correcting codes.

Invariant theorems in the Euclidean geometry with respect to conics

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A research is usually preceded by a natural aspiration of people to find new knowledge on the base of well-known facts. A large number of scientific facts are known for centuries, however new details and peculiarities are discovered in connection with them due to contemporary technologies. Information technologies based on specialized software propose rich possibilities for the development of new thinking. The software itself together with abundant arsenal of knowledge and skills for their effective application initiates specific way of thinking. Thus, the dynamic geometric software turns out to be a basic instrument to study the objects that belong to the classic Euclidean geometry. Various software products exist to perform corresponding investigations in the domain under consideration. The present talk uses the possibilities of the dynamic geometric software "THE GEOMETERS SKETCHPAD" (GSP) in the generalization of some classic but also of some not very popular theorems from the triangle geometry. The generalizations are realized in analyzing basic properties of the geometric objects under study.

The challenge to find a generalization of a geometric theorem is connected with a deep understanding of the considered figure properties. A necessary step is to clarify the relation among the elements of a given configuration, thus extracting the properties which could be changed. How to perform the change? Which elements and properties should be modified in order to change the corresponding theorem itself? The GSP program turns out to be a useful instrument in the process of answering these questions. The theorems included in the talk are mostly from the triangle geometry and are connected with different classes of circles, lines and points in the plane of that triangle. After the analysis of the corresponding relations the circles, the lines and the points are replaced by suitable conics, lines and points, thus keeping the validity of the theorems in the new situation. A deep knowledge of conic properties and constructive skills are necessary for the purpose. The program GSP is applied for fast elimination of various conjectures which turn out to be false, but also for the creation of convincing configurations leading to the

formulation of the desired assertions. The assertions themselves should be considered as true only in case they are strictly proven. Generalizations are obtained in many cases but reasons are found very often to reject some.

The established generalizations propose a new view on well-known geometric theorems and expose a deeper sense of the participating figures. They give possibilities to overcome the limits of previous perceptions. Thus, a gradual deepening of the understanding concerning the projective properties of conics is realized. Experience is obtained to discover certain theorems, which helps further investigations making them easier.

G-type spaces of ultradistributions over \mathbb{R}^d_+

STEVAN PILIPOVIĆ

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This lecture is devoted to the G-type spaces i.e. the spaces $G^{\alpha}_{\alpha}(\mathbb{R}^d_+)$, $\alpha \geq 1$ and their duals which can be described as analogous to the Gelfand-Shilov spaces and their duals but with completely new justification of obtained results. The Laguerre type expansions of the elements in $G^{\alpha}_{\alpha}(\mathbb{R}^d_+)$, $\alpha \geq 1$ and their duals characterise these spaces through the exponential and sub-exponential growth of coefficients. We provide the full topological description and by the nuclearity of $G^{\alpha}_{\alpha}(\mathbb{R}^d_+)$, $\alpha \geq 1$ the kernel theorem is proved.

As application, a class of the Weyl pseudo-differential operators is extended to one with the radial symbols with the exponential and sub-exponential growth rate.

Joint work with Smiljana Jakšić and Bojan Prangoski.

Some remarks on the recent history of the partial differential equations

Luigi Rodino

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We may fix as starting point of the recent history of the Partial Differential Equations the book of Courant and Hilbert "Methods of Mathematical Physics", Vol.II, representing an encyclopedia on known facts about PDE's up to 1940, with emphasis on equations of Mathematical Physics. Following the critique of Hadamard to the Cauchy-Kovalevskaya theorem, the need of a precise functional setting then emerges. This leads to the definition of Sobolev spaces and Schwartz distributions (1950-60). We survey some subsequent results on the general theory of PDE's (1970-80-90), as in the Hormander's work, and on the recent return (2000-2010) to other equations of Mathematical Physics, as Schrodinger and Boltzmann equations.

On the concept of connectedness

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The definition of connectedness is given in the beginning of 20th century by Riesz and Hausdorff.

The definition given earlier by Cantor in the period from 1879 until 1884 is:

A space is connected if for any two points x and y and any r > 0 there is a finite number of points $x = x_0, x_1, x_2, \ldots, x_n = y$ such that $d(x_i, x_{i+1}) < r$.

The two definitions coincide in the case of compact metric spaces.

Here are reformulations of Cantor definition of connectedness:

- 1) A space is *connected* if for any two points x and y and any r > 0 there is a finite chain of r-balls from x to y.
- 2) A space is *connected* if for any two points x and y and any open covering of X there is a finite chain of members of the covering from x to y.

The reformulation 2) coincides with the usual Riesz-Hausdorff definition for all topological spaces. This definition has advantages in some situations, for example a simpler definition of qasicomponents.

Theory of shape was introduced by K. Borsuk at the end of sixties of 20th century, as a better tool than homotopy theory for study of spaces with bad local properties. In that time the notion of *pointed 1-movability* (*strong connectedness*) was introduced also by Borsuk. The original definition uses embeddings of compact metric spaces in Hilbert cube.

Intrinsic definition of shape uses only coverings of a space and this is the reason why definition of connectedness only by coverings is the appropriate one and corresponds to intrinsic approach.

In the light of intrinsic approach the notion of pointed 1-movability will be reformulated by use only of coverings of the space and will be explained why the notion deserves name strong connectedness. It will be compared the notion of path connectedness in homotopy theory, the notion of connectedness in shape theory and the notion of strong connectedness (pointed 1-movability) in strong shape theory.



Algebra, Discrete Mathematics, Combinatorics and Number Theory

Groups and loops partitioned by subgroups

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A set of subgroups of a group is said to be a partition if every nonidentity element belongs to one and only one subgroup in this set. The study of groups with partition dates back to a paper by Miller published in 1906. In this presentation we will talk about groups and loops that are partitioned by subgroups. We will also discuss finite *p*-groups such that a subset of their maximal subgroups form an equal quasi-partition.

Characteristic of tiling generated by a root of polynomial

 $P(x) = x^4 - 3x^3 + x^2 - 2x - 1$

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There are different authors that have extensive studies on the tiling generated by Pisot numbers related to Pisot of degree 3. The connection of Pisot dual tailings play an important role on $\beta-$ expansion, it is often referred to Rényi as the first occurrence of $\beta-$ expansions. The theory of $\beta-$ expansion create a link between symbolic dynamics and a part of number theory. There exists a Pisot whose dual tiles are disconnected. Let β be a Pisot which is the root of equation $x^4-3x^3+x^2-2x-1=0$. At least one of the tiles is not connected. We try to give same property for the tiling generated by β .

The edge Folkman number $F_e(3,3;4)$ is greater than 19

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We denote by $\mathcal{H}_e(3,3;q)$ the set of graphs which have the property that in every coloring of their edges in two colors there is a monochromatic triangle and they do not contain a complete subgraph on q vertices K_q . The minimum number of vertices of graphs in $\mathcal{H}_e(3,3;q)$ is denoted by $F_e(3,3;q)$ and it is called an edge Folkman number. Folkman proved in [1] that $F_e(3,3;q)$ exists if and only if $q \geq 4$. It is well known that $F_e(3,3;q) = 6$ if $q \geq 7$. We also know that $F_e(3,3;6) = 8$ [2] and that $F_e(3,3;5) = 15$. The upper bound $F_e(3,3;5) \leq 15$ is proved in [4] and the lower bound is obtained in [5] with the help of computer. The exact value of the number $F_e(3,3;4)$ is not known. For now we know that this number is between 19 and 786, [6] and [3]. By improving the algorithms used in [6] we obtain the new bound $F_e(3,3;4) \geq 20$.

REFERENCES

- [1] J. Folkman, *Graphs with monochromatic complete subgraph in every edge coloring*, SIAM J. Appl. Math. 18, (1970) 19–24.
- [2] R. L. Graham, On edgewise 2-colored graphs with monochromatic triangles containing no complete hexagon, J. Combin. Theory, 4:300, 1968.
- [3] A. Lange, S. Radziszowski, X. Xu, *Use of MAX-CUT for Ramsey Arrowing of Triangles*, http://arxiv.org/abs/1207.3750, Submitted, 2012.
- [4] N. Nenov, An example of a 15-vertex Ramsey (3,3)-graph with clique number 4, C. A. Acad. Bulg. Sci., 34, (1981) 1487–1489, (in Russian).
- [5] K. Piwakowski, S. Radziszowski, S. Urbański, Computation of the Folkman number $F_e(3,3;5)$, J. Graph Theory, 32, (1999) 41–49.
- [6] S. Radziszowski, X. Xiaodong, *On the Most Wanted Folkman Graph*, Geombinatorics, XVI(4), (2007) 367–381.

An algebraic representation of a class of homogeneous Steiner quadruple systems

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A Steiner system S(t,k,v) is a pair (Q,\mathcal{B}) , where Q is a v-element set and \mathcal{B} is a collection of its k-element subsets (called blocks), such that every t-element subset of Q is contained in exactly one block. Steiner systems S(2,3,v) are known as Steiner triple systems (briefly STS) and their algebraic representatives are the idempotent totally symmetric quasigroups. Steiner systems S(3,4,v) are called Steiner quadruple systems (SQS). Algebraic structures which correspond to them are the idempotent totally symmetric ternary quasigroups.

For a given Steiner quadruple system (Q, \mathcal{B}) and $a \in Q$, by taking the set $Q \setminus \{a\}$ and the blocks $\{\{x,y,z\}\{x,y,z,a\} \in \mathcal{B}\}$, a Steiner triple system is obtained. It is called a derived triple system (DTS) of the quadruple system (Q, \mathcal{B}) . An SQS is called homogeneous if all of its derived triple systems are isomorphic. If all derived triple systems of an SQS are pairwise nonisomorphic, then the quadruple system is called heterogenous.

In this paper sufficient conditions for SQS to be homogenous are given, resulting with an algebraic representation of one class of homogenous quadruple systems.

Vector valued hyperstructures

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Basic concepts of vector valued hyperstructures, i.e. (n, m)-hyperstructures, are introduced. Namely, let $[\]$ be a mapping $[\]: H^n \to (\mathcal{P}^*(H))^m$ from the n-th cartesian product of H to the m-th cartesian product of $\mathcal{P}^*(H)$, where $\mathcal{P}^*(H)$ is the set of all nonempty subsets of H. Then $[\]$ is called an (n, m)-hyperoperation on H or, if it is not necessary to emphasize the integers n and m, then we will say that $[\]$ is a *vector valued hyperoperation* instead of (n, m)-hyperoperation. We can associate to the operation $[\]$ a sequence of m n-ary hyperoperations $[\]_s: H^n \to \mathcal{P}^*(H)$, $s \in \{1, 2, \ldots, m\}$, by putting

$$[a_1 \ldots a_n]_s = B_s \Leftrightarrow [a_1 \ldots a_n] = (B_1, \ldots, B_m),$$

for all $a_1, ..., a_n \in H$. Then, we call $[\]_s$ the *s-th component hyperoperation* of $[\]$ and write $[\] = ([\]_1, ..., [\]_m)$.

An algebraic structure $\mathbf{H} = (H, [\])$, where $[\]$ is an (n, m)-ary hyperoperation defined on a nonempty set H, is called an (n, m)-hypergroupoid or vector valued hypergroupoid.

We define (i, j)-associative and weekly (i, j)-associative (n, m)-hyperoperation, (n, m)-hypersemigroup and week (n, m)-hypersemigroup, (n, m)-hypergroup and week (n, m)-hypergroup, cancellative, partially cancellative and strongly cancellative (n, m)-hypergroupiod, weak neutral element and neutral element. Some of their properties are investigated for $n = m + k, k \ge 1$ and many examples are given.

Categorical syllogisms on Heyting algebras

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In this work, our aim is to construct a system, which is an Heyting algebra, for examining categorical syllogisms. For this aim, we define categorical syllogisms via diagrammatic method, which gives us a suitable treatment to logical reasoning with Carolls diagrams, for representations of the fundamental Aristotelian categorical propositions and show that they are closed under the syllogistic feature of inference which is the deletion of middle term. Thus, it is implemented to let the formalism comprise simultaneously bilateral and trilateral diagrammatical appearance and an uncomplicated algorithmic nature.

In this regard, we represent the quantitative relations between syllogisms terms by means of bilateral and trilateral diagrams. Finally, we examine valid forms of syllogisms using algebraic methods, and we construct an Heyting algebra for categorical syllogisms by using sets.

REFERENCES

- [1] L. Caroll, Symbolic Logic, Clarkson N. Potter, 1896.
- [2] J. Łukasiewicz, Aristotles Syllogistic From the Standpoint of Modern Logic, Clarendon Press, Oxford, 1951.
- [3] S. Burris, H.P. Sankappanavar, A Course in Universal Algebra, Springer-Verlag, 1981.
- [4] İ. Şentürk, Tahsin Öner, U. Nuriyev, An Algebraic Approach to Categorical Syllogisms By Using Bilateral Diagrams, Theoretical and Applied Aspects of Cybernetics, Proceedings of the 5th International Scientific Conference of Students and Young Scientists (Kyiv-Ukraine) (2015) 14-21.

SECTION 2

Complex Analysis, Real Analysis and Functional Analysis

Approximation for periodic functions via weighted A-statistical convergence for sequences of positive linear operators

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In this paper, we use the notion of weighted A-statistical convergence to prove the Korovkin approximation theorem for the functions 1, cos and sin in the spaces of all continuous 2π -periodic functions on the real line. We also study the rate of weighted A-statistical convergence for periodic functions.

REFERENCES

- [1] F. Altomare, *Korovkin-type theorems and approximation by positive linear operators*, Surv. Approx. Theory 5 (2010) 92–164.
- [2] N. L. Braha, H. M. Srivastava, S. A. Mohiuddine, A Korovkins type approximation theorem for periodic functions via the statistical summability of the generalized de la Vall'ee Poussinmean, Applied Mathematics and Computation, vol. 228 (2014) pp. 162–169.
- [3] O. H. H. Edely, S. A. Mohiuddine, A. Noman, *Korovkin type approximation theorems obtained through generalized statistical convergence*, Applied Mathematics Letters, vol. 23, no.11 (2010) pp. 1382–1387.
- [4] M. Mursaleen, V. Karakaya, M. Ertürk, F. Gürsoy, Weighted statistical convergence and its application to Korovkin type approximation theorem, Applied Mathematics and Computation, vol. 218, no. 18 (2012) pp. 9132–9137.

Remark about characterization of 2-inner product

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Characterization of the 2-inner product is the focus of interest of many mathematicians. In this paper, proofs of two characterizations of 2-inner product, which are actually consequences of the Theorem 1 ([3]), are given. Generalizations of already know Hayashi (see [1], pg. 297) and Zarantonello ([2]) inequalities are fully elaborated as well.

- [1] D. S. Mitrinović, J. E. Pečarić, V. Volenec, Recent Advances in Geometric Inequalities, Kluwer Academic Publisher, 1989.
- [2] E. H. Zarantonello, Projections on convex sets in Hilbert spaces and spectral theory, Contributions to Nonlinear Functional Analysis, Acad. Press., New York, 1971, 237–424.
- [3] S. Malčeski, K. Anevska, R. Malčeski, *New characterization of 2-pre-Hilbert Space*, Matematički bilten, Vol. 38(XLIV), No. 1 (2014), 31–38.

On the theory of differential subordination and superordination

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The lecture deals with some recent results obtained by the author involving different classes of analytic differential and integral operators that preserve the subordination and the superordination. Using the technique of *admissible functions* introduced by P. T. Mocanu and S. S. Miller, we obtained *sandwich-type* and some general *averaging* (or mean-value) operators. These new results extend a few of the previous classical results belonging to different authors, and moreover, some interesting applications obtained for appropriate choices are also given.

Thus, using the method given in [1], we gave some recent results about different classes of analytic integral operators that preserve the subordination and the superordination, in order to obtain *sandwich-type* and *modified* (*weighted*) *sandwich-type* results.

We obtained subordination and superordination preserving properties for the Saigo type generalized fractional differ-integral operator, defined for multivalent functions in the open unit disk [2]. Secondly, we give subordination and superordination preserving properties for the new defined generalized operator involving the *Srivastava-Attiya integral operator*; differential sandwich-type theorems for these univalent functions, and some consequences involving well-known special functions are also presented [3].

Further, we obtained the subordination and superordination preserving properties of certain nonlinear integral operators defined on the space of normalized analytic functions in the open unit disk, and the sandwichtype theorems for these integral operators are also presented [4]. Finally, we gave subordination, superordination and sandwich-preserving new theorems for certain integral operators that generalize some recently ones [5].

- [1] Bulboacă, T., *Sandwich-Type Results for a Class of Convex Integral Operators*, Acta Math. Sci. Ser. B Engl. Ed., 32(3)(2012), 989–1001.
- [2] Prajapat, J. K., Bulboacă, T., Double Subordination Preserving Properties for Generalized Fractional Differ-integral Operator, J. Class. Anal., 5(2)(2014), 147–161.
- [3] Prajapat, J. K., Bulboacă, T., Double Subordination Preserving Properties for a New Generalized Srivastava-Attiya Integral Operator, Chinese Ann. Math. Ser. B, 33(4)(2012), 569–582.
- [4] Cho, N. E., Bulboacă, T., Srivastava, H. M., A General Family of Integral Operators and Associated Subordination and Superordination Properties of Some Special Analytic Function Classes, Appl. Math. Comput., 219(2012), 2278–2288.
- [5] Aouf, M. K., Bulboacă, T., Seoudy, T. M., Certain Family of Integral Operators Preserving Subordination and Superordination, Acta Math. Sci. Ser. B Engl. Ed., 34(4)(2014), 1–13.

Asymmetric inner product and asymmetric quasi norm function

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Semi-inner products, that can be naturally defined in general Banach spaces over the real or complex number field, play an important role in describing the geometric properties of these spaces.

In the last forty years a large number of authors have used them as a powerful tool in investigating various properties.

Starting from its axiomatic, many researchers have made various modifications passing in its generalization.

Semi-scalar products mark the very first generalizations of the scalar product function. The strong bond between these functions with the norm function has made it possible to obtain a lot of interesting results which are connected with the orthogonality and convexity.

This paper attempts to generalize the semi scalar product concept according to G. Lumer by replacing Cauchy inequality with another inequality which is more generalized. Based on this attempt of generalization it is built a function which fulfills the conditions which are changed. In this paper it is also generalized quasi norm function by replacing homogeneity condition with a more restricted condition by producing this time a more generalized asymmetric semi norm function. As a result, in this paper it is defined the asymmetric inner product function and the asymmetric quasi norm function. Moreover, it is even given relation between these two.

On quasiconformal selfmappings of the unit disk and the unit ball

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We will see the Lipschitz continuity of quasiconformal selfmappings of the unit disk, properties of harmonic quasiconformal quasi-isometries and the co- Lipschicity of harmonic quasiconformal selfmappings of the unit disk at the origin. We will mention also harmonic quasiconformal selfmappings of the unit ball.

Approximation by: Riemann zeta-function; polynomials (rational functions) with constrained zeros (poles)

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Andersson showed than an improvement of the spectacular theorem of Voronin on the universality of the Riemann zeta-function is equivalent to a natural problem on polynomial approximation with constrained zeros. Classical approximation is by rational functions with constrained poles. We consider meromorphic approximation with constrained poles on Riemann surfaces, bearing in mind that poles and zeros have a similar nature for meromorphic functions. Any new results are jointly with Fatemeh Sharifi.

Sequentially convergent mappings and common fixed points of mappings in 2-Banach space

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In the past few years, the classical results about the theory of fixed point are transmitted in 2-Banach spaces, defined by A. White (see [3] and [8]). Several generalizations of Kannan, Chatterjea and Koparde-Waghmode theorems are given in [1], [4], [5] and [7]. In this paper, several generalizations of already known theorems about common fixed points of mappings in 2-Banach spaces are proved, by using the sequentially convergent mappings.

- [1] A. Malčeski, A. Ibrahimi, R. Malčeski, Extending Kannan and Cratterjea theorems in 2-Banach spaces by using sequentially convergent mappings, (private communication)
- [2] A. Malčeski, S. Malčeski, K. Anevska, R. Malčeski, New Extension of Kannan and Chatterjea Fixed Point Theorems on Complete Metric Spaces, British Journal of Mathematics & Computer Science, Vol. 17, No. 2, (2016), 1–10
- [3] A. White, 2-Banach Spaces, Math. Nachr. Vol. 42 (1969), 43–60
- [4] M. Kir, H. Kiziltunc, Some New Fixed Point Theorems in 2-Normed Spaces, Int. Journal of Math. Analysis, Vol. 7 No. 58 (2013), 2885–2890
- [5] P. Chouhan, N. Malviya, *Fixed Points of Expansive Type Mappings in 2-Banach Spaces*, International Juornal of Analysis and Applications, Vol. 3 No. 1 (2013), 60–67
- [6] P. K. Hatikrishnan, K. T. Ravindran, Some Properties of Accretive Operators in Linear 2-Normed Spaces, International Mathematical Forum, Vol. 6 No. 59 (2011), 2941–2847
- [7] R. Malčeski, A. Ibrahimi, On Contraction Mappings and Fixed Point in 2-Banach Spaces, (in print)
- [8] R. Malčeski, K. Anevska, *About the 2-Banach spaces*, International Journal of Modern Engineering Research (IJMER), Vol. 4 Iss. 5 (2014), 28–32

Extending Kannan and Chatterjea theorems in 2-Banach spaces by using sequentially convergent mappings

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The term of 2-normed space is given by S. Gähler ([7]), and 2-banach space by A. White ([1]). Further, in [3], P. K. Hatikrishnan, K. T. Ravindran, have defined a contractive mapping in 2-normed space. In [2] and [9], generalizations of Kannan ([4]) and Chatterjea ([8]) fixed points theorems in 2-Banach spaces are given. In this paper by using sequentially convergent mappings, the results given in [2] and [9] are generalized in 2-Banach spaces. The latter might also be considered as generalizations of the results given in [10].

- [1] A. White, 2-Banach Spaces, Math. Nachr. Vol. 42 (1969), 43–60
- [2] M. Kir, H. Kiziltunc, *Some New Fixed Point Theorems in 2-Normed Spaces*, Int. Journal of Math. Analysis, Vol. 7 No. 58 (2013), 2885–2890
- [3] P. K. Hatikrishnan, K. T. Ravindran, *Some Properties of Accretive Operators in Linear 2-Normed Spaces*, International Mathematical Forum, Vol. 6 No. 59 (2011), 2941–2847
- [4] R. Kannan, Some results on fixed points, Bull. Calc. Math. Soc. Vol. 60 No. 1, (1968), 71–77
- [5] R. Malčeski, K. Anevska, *About the 2-Banach spaces*, International Journal of Modern Engineering Research (IJMER), Vol. 4 Iss. 5 (2014), 28–32
- [6] S. Banach, Sur les operations dans les ensembles abstraits et leur application aux equations intégrales, Fund. Math. 2 (1922), 133–181
- [7] S. Gähler, Lineare 2-normierte Räume, Math. Nachr. 28 (1965), 1–42
- [8] S. K. Chatterjea, Fixed point theorems, C. R. Acad. Bulgare Sci., Vol. 25 No. 6 (1972), 727–730
- [9] R. Malčeski, A. Ibrahimi, Contraction Mappings and Fixed Point in 2-Banach Spaces, (in print)
- [10] A. Malčeski, S. Malčeski, K. Anevska, R. Malčeski, New Extension of Kannan and Chatterjea Fixed Point Theorems on Complete Metric Spaces, British Journal of Matematics & Computer Science, Vol. 17, No. 2, (2016), 1–10

Some characterizations of 2-inner product

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The characterization of 2-inner product is an issue which is in focus of interest of many mathematicians. In this paper, several equivalent characterizations of 2-inner product, that are consequences of Theorem 2 ([1]) are discussed. Thus, the equivalence of generalizations of the Jordan and von Neumann ([2]) and also Frechet ([3]) classical results, are proven. Furthermore, the characterization of Hlawka, the characterization of D. S. Marinescu, M. Monea, M. Opincariu and M. Stroe ([4]) are proven as well.

- [1] R. Malčeski, K. Anevska, *Characterization of 2-inner product by strictly convex 2-norm of modul c*, International Journal of Mathematical Analysis, Vol. 8, no. 33 (2014), 1647–1652.
- [2] M. Fréchet, Sur la definition axiomatique dune classe despaces vectoriels distanciés applicable vectorillement sur lespace de Hilbert, Ann. of Math. Vol. 36 (3) (1935), 705–718.
- [3] P. Jordan, J. von Neumann, *On inner products in linear, metric spaces*, Ann. of Math. Vol. 36 (3) (1935), 719–723.
- [4] D. S. Marinescu, M. Monea, M. Opimcariu, M. Stroe, Some Equivalent Characterizations of Inner Product Spaces and Their Consequences, Filomat 29 (7) (2015), 1587–1599.

A characterization of strictly convex 2-normed space

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The terms of 2-norm and 2-normed space are given by S. Gähler in the paper [1]; the term of strictly convex 2-normed spaces is given by Ch. Diminnie, S. Gähler and A. White ([2]). In this paper a few characterizations of strictly convex 2-normed spaces are obtained.

- [1] S. Gähler, Lineare 2-normierte Räume, Math. Nachr., Vol. 28 (1965), 1-42.
- [2] C. Diminnie, S. Gähler, A. White, 2-Inner Product Spaces, Demonstratio Mathematica, Vol. VI, No 1 (1973), 525–536.

Recent results on a class of univalent functions

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Let \mathcal{A} denote the family of all analytic functions in the unit disk \mathbb{D} := $\{z \in \mathbb{C} : |z| < 1\}$ with the normalization f(0) = 0 = f'(0) - 1. Let \mathcal{U} denote the set of all $f \in \mathcal{A}$ in \mathbb{D} satisfying the condition

$$\left| \left(\frac{z}{f(z)} \right)^2 f'(z) - 1 \right| < 1 \text{ for } z \in \mathbb{D}.$$

It is well-known that $\mathcal U$ belongs to the class of univalent functions in $\mathbb D$. In this lecture we give some recent results concerning the class $\mathcal U$ given by the author and his cooperators.

Statistical extensions of some classical Tauberian theorems for Cesàro summability of triple sequences

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Tauberian theorems for Cesàro summability method were firstly obtained by Tauber. Tauber proved that necessary condition for the convergence of single sequences which are Cesàro summable is $n\Delta_n u_n = o(1)$. However, this condition was reduced to the two-sided boundedness condition by Hardy. Thereafter studies done related to this subject were concentrated on the weakening of condition given by Tauber.

In this paper, we show that the conditions under which *P*-convergence follows from the statistical convergence which behave as a summability method for triple sequences. In section 2, we present some lemmas which will be used in the proofs of our main theorems. In section 3, we prove that a statistically convergent triple sequence is P-convergent under two-sided boundedness conditions and slowly oscillating conditions in certain senses. In last section, we show that the (C, 1, 1, 1) means of a statistical convergent triple sequence is also statistical convergent under the boundedness condition of the sequence and we also give some classical Tauberian theorems for a triple sequence that *P*-convergence follows from statistically (*C*, 1, 1, 1) summability under the two-sided boundedness conditions and slowly oscillating conditions in certain senses. In the last theorem of this section, we obtain the statistical convergence of a sequence from its statistical (C, 1, 1, 1) summability under weaker conditions. All of these theorems demonstrate that some theorems given by Móricz and Schoenberg are also valid for triple sequences.

On modified Bleimann, Butzer and Hahn approximation operators

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In this work, we consider sequences of Bleimann, Butzer and Hahn operators which are based on a function τ . This function is a continuously differentiable function on \mathbb{R}^+ , such that $\tau(0)=0$, inf $\tau(x)\geq 1$. We give a Korovkin-type theorem and prove uniform approximation of the generalized Bleimann, Butzer and Hahn operator. We also investigate some monotonicity properties of the operator.

Properties of a tiling generated by the golden ratio

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Let β be a Pisot number, that is an algebraic integer greater than 1 whose conjugates other than itself have modulus smaller than 1. There are certain criterions of finiteness for representing real numbers , by a greedy algorithm, in a Pisot number base. In this paper we will show which is the digit expansion of number 1 in base $\beta = (1+\sqrt{5})/2$ and define the admissibility condition for the other numbers to be represented in that base. Then we will construct a tiling of the real line by the digit expansion in base β and will show that it is selfsimilar. Also we will show that the central tile is intersected precisely with the tile $K._1$ and $K._{01}$ and will find the point of their intersection. Then our aim is to study in general the properties of tiling generated by Pisot number of second order.

Some results about a filtration of starlike functions

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joint work with David Shoikhet and Mark Elin

Let \mathcal{A} be the class of functions f that are analytic in the open unit disk Δ and are normalized such that f(0) = f'(0) - 1 = 0. Also, let \mathcal{S}^* be the class of normalized starlike univalent functions

$$\mathcal{S}^* = \left\{ f \in \mathcal{A} : \operatorname{Re}\left[\frac{zf'(z)}{f(z)}\right] > 0, \ z \in \Delta \right\}.$$

Now, using the operator

$$D[f](t,z) = t|z|^2 \frac{f(z)}{zf'(z)} + (1-t)\left[1 - \frac{f(z)f''(z)}{[f'(z)]^2}\right] (1-|z|^2)$$

(t is real and $z \in \Delta$) we define class \mathcal{S}_t^* , $t \in [0,1]$, consisting of functions $f \in \mathcal{A}$ such that

$$\operatorname{Re} D[f](t,z) \ge 0, \quad z \in \Delta.$$

It turns out that the family of classes S_t^* , $t \in [0,1]$, is a filtration of the class of starlike functions. In addition, for a function f from S_t^* we give a result over the real part of $\frac{f(z)}{zf'(z)}$ and an approximation property of f.

Common fixed points in b-dislocated metric spaces using (E.A) property

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In this paper, we prove coincidence and common fixed point results for one and two pairs of mappings that satisfy the (E.A) property and its generalized variants in the framework of b-dislocated metric spaces. Our results generalizes and extend some existing results in the literature.

- [1] C. T. Aage, J. N. Salunke, The results on fixed points in dislocated and dislocated quasimetric space, Appl. Math. Sci.,2(59), (2008), 2941-2948.
- [2] Aamri M. and Moutawakil D. El., Some new common fixed point theorems under strict contractive conditions, J. Math. Anal, Appl. 270, 181-188, 2002.
- [3] A. Beiranvand, S. Moradi, M. Omid, H. Pazandeh, Two xed point theorems for special mapping, arXiv:0903.1504v1 [math.FA].
- [4] I. A. Bakhtin, *The contraction mapping principle in quasimetric spaces*, Funct. Anal., Unianowsk Gos. Ped. Inst. 30, (1989), 26-37
- [5] P. Hitzler, A. K. Seda, Dislocated topologies, J. Electr. Engin, 51(12/S):3:7, 2000.
- [6] N. Hussain, J. R. Roshan, V. Parvaneh and M. Abbas, *Common fixed point results for weak contractive mappings in ordered b-dislocated metric spaces with applications*, Journal of inequalities and Applications, 1/486, (2013)
- [7] E. Karapinar and P. Salimi, *Dislocated metric space to metric spaces with some fixed point theorems*, Fixed point theory and applications (2013) 2013:222
- [8] S. Czerwik, *Contraction mappings in b-metric spaces*, Acta Math. Inform. Univ. Ostrav. 1, 5-11 (1993)

SECTION 3 Differential Equations and Dynamical Systems

About the zeros and the oscillatory character of the solution of one areolar equation of second order

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In the paper, one linear areolar equation of second order with constant coefficients is considered, regarding the zeroes and the oscillatory character of its general solution. In this equation the first derivative is missing. Some theorems will be proven and some examples will be given for different cases of the coefficients.

On the types of linear differential equations of second order and their solutions

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In this paper, types of linear differential equations of second order are defined according to the known classification of the types of linear partial differential equations of second order. Then the shapes of their general solutions are written and some classes of equations are solved.

Strongly isochronous centers of two-dimensional cubic systems with degenerate infinity

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The maximum order of strong isochronous center of two-dimensional cubic system with degenerate infinity and its initial polar angle computed is provided in this paper.

Second order dynamic operator with transmission conditions and associated contractive operator

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In this paper, we investigate the spectral properties of a boundary value transmission problem generated by a dynamic equation on the union of two time scales. For such an analysis we assign a suitable dynamic operator which is in limit-circle case at infinity. We also show that this operator is a simple maximal dissipative operator. Constructing the inverse operator we obtain some information about the spectrum of the dissipative operator. Moreover, using the Cayley transform of the dissipative operator we pass to the contractive operator which is of the class C_0 . With the aid of the minimal function we obtain more information on the dissipative operator. Finally, we investigate other properties of the contraction such that multiplicity of the contraction, unitary colligation with basic operator and CMV matrix representation associated with the contraction.

Spectral properties of eigenvalues and eigenfunctions of a discontinuous boundary value problem with retarded argument

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In this study a discontinuous boundary value problem with retarded argument and with transmission conditions at the point of discontinuity are investigated. In the special case that our problem is continuous (i.e. when $\delta=1$ in below) the obtained results coincide with the corresponding results in [1].

REFERENCES

[1] S. B. Norkin, *On Boundary Problem of Sturm-Liouville Type for Second Order Differential Equation With Retarded Argument*, Izv. Vyss., Ucebn Zaved Matematika 6(7) (1958) 203–214 (in Russian).

Approximate method for the initial value problem for impulsive difference equations

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An algorithm for constructing two monotone sequences of upper and lower solutions of the initial value problem for nonlinear impulsive difference equations is given. It is proved that both functional sequences are convergent and their limits are minimal and maximal solutions of the considered problem.

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Extended discrete transformation method for nonlinear systems of ordinary and fractional differential equations

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We present an extension of the discrete transformation method for numerical solution of nonlinear oscillatory, possibly chaotic systems, of both ordinary and fractional differential equations. The method is applied at each integrating timestep by constructing a local solution in a form of a truncated series, which are power series in the case of ordinary differential equations, and fractional series in the case of fractional differential equations. To illustrate the numerical approach, we analyze several examples of chaotic oscillatory systems of differential equations of both ordinary and fractional type, and provide a detailed bifurcation analysis of the solutions depending on the system parameters. We also make a comparative analysis of the proposed method with some standard methods for numerical solution of differential equations.

- [1] J. K. Zhou, Differential transformation and application for electrical circuits, Huazhong University Press, Wuhan, China, 1986.
- [2] A. Arikoglu, I. Ozkol, Solution of boundary value problems for integro-differential equations by using transform method, Appl. Math. Comput. 168, (2005) 1145 1158 .
- [3] V. S. Ertrurk, S. Momani, Solving systems of fractional differential equations using differential transform method, J. Comp. Appl. Math. 215, (2008) 142 151.

Multistationarity in reaction networks models

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Mathematical models in biology are often available in the form of ordinary differential equations. Multistationarity or the existence of several steady states is an ubiquitous phenomenon in biochemical reaction models. We focus on providing an algorithm for the existence of several positive steady states that can be easily applied. The input is a system of ordinary differential equations whose behavior depends on a set of parameters and the output is a single polynomial. The sign of this polynomial decides the capacity for multiple (unique) steady states. Often we can obtain parameter inequalities that ensure the sign resulting in multiple (unique) steady states that the parameter inequalities in turn define parameter regions where multiple (unique) steady states exist.

This is a joint work with Carsten Conradi, Elisenda Feliu and Carsten Wiuf.

On the regularized trace formula of the second order differential operator equation with semi-periodic boundary conditions

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In this study we investigate the spectrum and regularized trace of the second order differential operator equation with semi-periodic boundary conditions.

For one system differential equations dual of the system of Lorenz

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In this paper the system of Lorenz and one of its dual systems differential equations are reviewed. Some of the important common properties of both systems are given.

On integrable extensions

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We present the integrability of q-difference equations in the light of existence of multi-soliton solutions. Moreover, we constitute a unifying framework for q-discrete equations and their multi-soliton solutions that comprises various q-difference type of soliton equations such as sine-Gordon, KdV and Toda equations. For this purpose, we present a generic equation and develop three-q-soliton solutions, which are expressed in the form of polynomials in power functions. We conjecture the nonexistence of other unifying approaches to study integrable equations on quantum numbers or on any time scales via Hirota perturbation.

On continuous dependence of solutions of dynamic equations

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The main goal of the talk is to present a new approach to the problem of continuous dependence of solutions of differential or dynamic problems on their domains. This is of particular interests when we use dynamic (difference, in particular) equations as discretization of a given one. We cover a standard construction based of difference approximations for the continuous one, but we are not restricted only to this case. For a given differential equation we take a sequence of time scales and we study the convergence of time scales to the domain of the considered problem. We choose a kind of convergence of such approximated solutions to the exact solution. This is a step for creating numerical analysis on time scales and we propose to replace in such a situation the difference equations by dynamic ones. In the proposed approach we are not restricted to the case of classical numerical algorithms. Moreover, this allows us to find an exact solution for considered problems as a limit of a sequence of solutions for appropriate time scales instead of solving it analytically or calculating approximated solutions for the original problems.

Approximation for the solutions of Lorenz system with systems of differential equations

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Using the systems of difference equations from [1] and [2] as approximation for the solutions of the Lorenz system of differential equations we obtain three new systems of differential equations whose locally behavior is close to the behavior of Lorenz system. By computer simulations we give examples where locally behavior of the systems is analyzed by comparing of behavior for the different time step. By increasing the time step it can be seen that behavior of systems is farther away from the behavior of Lorenz system.

SECTION 4

Topology, Geometry, Differential Geometry and Mathematical Physics

Birman-Hilden property and Non-orientable surfaces

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A (branched) covering between surfaces is said to have or satisfy the Birman–Hilden property, if the finite index subgroup of the mapping class group of the base surface, injectively maps to the mapping class group of the covering surface modulo a finite subgroup. This property is studied for regular, irregular, branched and unbranched coverings of orientable surfaces by some mathematicians before.

In this talk, we will give one sufficient condition for coverings between non-orientable surfaces to satisfy Birman–Hilden property.

Analysis of curves with Lie groups

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Investigating curves and surfaces are forming very important part of differential geometry since the 18th century. Besides this, at the late 19th century, differentiable manifolds have arisen. And the Lie groups are important structures because of being a bridge from algebraic properties to geometric properties of manifolds. In this study, we analyse the some special curves which are given by a new adjoint approach based on differential geometry by Wang ([1]). While analysing these curves we used the Lie groups and relations between Lie groups which are given by Karger ([2]).

- [1] D. L. Wang, J. Liu, D. Z. Xiao, Kinematic differential geometry of a rigid body in spatial motion-I. A new adjoint approach and instantaneous properties of a point trajectory in spatial kinematics, Mech. and Mach. Theory, Vol 32, No 4 (1997) pp. 419-432.
- [2] A. Karger, J. Novak, *Space Kinematics and Lie Groups*, Gordon and Breach Science Publishers, Switzerland, 1985.

Theorems of metrizations in $(3, j, \rho)$ - \mathcal{B} -metrizable spaces, $j \in \{1, 2\}$

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For a given (3,2)-metric d on M, we show that every (3,2)- \mathcal{B} -metrizable space, for $\mathcal{B} = \{N, S, K\}$, is metrizable and topological space (M, τ) is metrizable if and only if it is (3,2)-N-D-metrizable.

Hypersurfaces of quaternionic projective space of Chen type 2

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In a curvature-adapted hypersurface M of a quaternionic-Kähler manifold \overline{M} the maximal quaternionic subbundle \mathcal{D} of TM and its orthogonal complement \mathcal{D}^{\perp} in TM are, by definition, invariant subspaces of the shape operator A at each point. We classify curvature-adapted real hypersurfaces M of the quaternionic projective space $\mathbb{H}P^m$ that are of Chen 2-type in an appropriately defined Euclidean space of quaternion-Hermitian matrices. That means that the position vector of such submanifold in this ambient Euclidean space of matrices is decomposable into a sum of a constant vector and two nonconstant vector eigenfunctions of the Laplace operator of the submanifold, belonging to different eigenspaces. In the quaternionic projective space they include all geodesic hyperspheres except one, two series of tubes about canonically embedded quaternionic projective spaces of lower dimensions and two particular tubes about canonically embedded $\mathbb{C}P^m \subset \mathbb{H}P^m$, which turn out to have certain extremal property with respect to stability. Except for these two tubes, other tubes about $\mathbb{C}P^m$ are mass-symmetric and of 3-type.

Convergence of sequences in $(3, j, \rho)$ -N-metrizable spaces, $j \in \{1, 2\}$

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In this paper we consider $(3, j, \rho)$ -N-metrizable spaces, $j \in \{1, 2\}$ in terms of convergence of sequences. We define four types of convergence in this spaces. We give examples which show that in general this four types of convergence do not coincide in $(3, 1, \rho)$ -N-metrizable spaces. We show that in (3, 2)-N-metrizable spaces these four types of convergence coincide.

On product of quasicomponents

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Due to simplicity we characterize quasicomponents by continuous mappings and we obtain the well known theorem that product of quasicomponents Q(x), Q(y) of topological spaces X, Y, respectively gives quasicomponent in the product space. If spaces X, Y are assumed to be locally-compact, paracompact and Haussdorf then we prove that space of quasicomponents of the product $Q(X \times Y)$ has equivalent topology with the product space $Q(X) \times Q(Y)$. Consequently, these two spaces have same topological properties.

Kinematic geometry of pseudo-spherical evolutes

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In this study, we define pseudo-spherical evolutes corresponding to the trajectory of a point in spherical kinematics. Also, we obtain general expressions for the curvature properties of the nth pesudo-spherical evolute in terms of the geodesic curvature and the derivative of the geodesic curvature for the (n-1)th pseudo-spherical evolute.

- [1] Schaaf, J. A. and Yang, A. T., Kinematic geometry of spherical evolutes, J. Mech. Des. 114 (1) (1992), 109–116.
- [2] Sato, T., Pseudo-spherical evolutes of curves on a spacelike surface in three dimensional Lorentz–Minkowski space, J. Geom. 103 (2012), 319–331.
- [3] Chiang, C.H., *Kinematics of spherical mechanisms*, Cambridge University Press, Cambridge, England, 1988.

Some characterizations of point-line trajectories in Lorentz 3-space

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This work deals with curvature theory of point-line trajectories in Lorentz 3—space. We present the characterization by indicatrix, directrix and their relationship in Lorentz 3—space. This characterization and relationship indicate a point-line trajectory.

- [1] O. Bottema, B. Roth, Theoretical Kinematics, Dover Publications, New York, 1990.
- [2] J. M. McCarthy, B. Roth, *The Curvature Theory of Line Trajectories in Spatial Kinematics*, ASME J. Mech. Des., 103 (1981) 718–724.
- [3] B. O'Neill, Semi-Riemannian Geometry with Applications to Relativity, Academic Press, London, 1983.
- [4] B. S. Ryuh, *Robot trajectory planning using the curvature theory of ruled surfaces*, Doctoral dissertation, Purdue University, West Lafayette, Ind, USA, 1989.
- [5] K. L. Ting, Y. Zhang, R. Bunduwongse, *Characterization and Coordination of Point-line Trajectories*, ASME Journal of Mechanical Design, 127 (2005) 502-505.

Equations of motion of classical non-relativistic strings and relativistic strings

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To formulate the dynamics of a system we can write either the equations of motion or, alternatively, an action. We construct an analytic model describing the macroscopic properties of non-relativistic string and relativistic string. We study the equations of motion for nonrelativistic strings and develop the Lagrangian approach to their dynamics. More importantly, the equations of motion for the relativistic string, the action is a natural generalization of the relativistic particle action. We use the proper area of this surface as the action; this is the Nambu-Goto action. We study the reparameterization property of this action, identify the string tension, and find the equations of motion.

Some examples of CalabiYau manifolds

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The space-time in string theory is often described by means of a mathematical object called manifold. Manifolds are very important objects from the mathematical and the physics point of view, not only in string theory. Calabi-Yau manifolds are complex manifolds and they exist in any even dimension. The simplest examples of Calabi-Yau manifolds have one complex dimension. Some simple examples of non compact Calabi-Yau two-folds, which have two complex dimensions are $\mathbb{C}^2 = \mathbb{C} \times \mathbb{C}$, $\mathbb{C} \times T^2$. K3 and T^4 are two examples of four-dimensional compact Kähler manifolds for which they exist. Examples of a Calabi-Yau n-folds can be constructed as a submanifold of $\mathbb{C}P^{n+1}$ for all n > 1. The most important examples are manifolds of G_2 and Spin(7) holonomy.

Noncommutative Kähler Manifolds and their Fock representations

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We introduce some kind of Fock representations of noncommutative Kähler manifolds and give their explicit expressions. In deformation quantization of Kähler manifolds with separation of variables formulated by Karabegov. Based on the theory, we construct the representation of noncommutative Kähler manifolds and give a dictionary to translate between the Fock like representations and functions on noncommutative Kähler manifolds concretely.

Connection between strong proximative and strong approximative sequences

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The required notions are given in [1] and [2]. In the presentation it is given a way how on continuous functions to associate V-continuous, and it is defined fuction from the classes of strong approximative sequences to the classes of strong proximative sequences.

- [1] Yu. Lisica, S. Mardešić, *Coherent homotopy and strong shape for compact metric spaces*, Glasnik Mat. Vol. 20(40) (1985), 159–167.
- [2] N.Shekutkovski, *Intrinsic definition of strong shape for compact metric spaces*, Topology Proceedinds 39 (2012), 27–39.

Application of the Geometry of Curves in 3-dimensional Euclidean Space

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In this article the so called spin velocities are studied, which were recently introduced by the author, using the geometry of curves. Some of their essential properties are given and they are rather different than the ordinary velocities. Indeed, the spin velocities are non-inertial, they are not constrained by the velocity of light c, instead of the Lorentz transformations for them the Galilean transformations should be used, and while in case of inertial velocities the components $\{F_{ij}\}$ of the electromagnetic field transform as a tensor with respect to the Lorentz transformations, in case of spin velocities the components $\{F_{ij}\}$ remain unchanged. Also, the method of calculation of the spin velocities, by using the curvatures and torsions of curves in 3-dimensional Euclidean space is given. Some important applications of the spin velocities are given as well.

On matrices and vector field in Minkowski 3-space

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In this work, we give three different forms for matrix A depending on the causel characters of the vector x by analyzing the non-zero solutions of the equation A(x) = 0, $x \in \mathbb{E}^3_1$, in Minkowski 3–space, where A is the skew-symmetric matrix corresponding to the linear map \mathbf{A} . Also, we give some theorems and classifications about integral curves of a linear vector field in Minkowski 3-space.

- [1] Acratalishian, A., On the Linear Vector Field in \mathbb{E}^{2n+1} , Commun. Fac. Sci. Ank., 39, (1989) 21–35.
- [2] Duggal, K. L., Bejancu, A., Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications, Kluwer Academic Publishers, The Nedherlands, 1996.
- [3] Ferrandez, A., Gimenez A., and Lucas P., Null Generalized Helices in Lorentz-Minkowski Spaces, J. Phys. A: Math. Gen., 35, (2002), 8243–8251.
- [4] Galbis A., Maestre M., Vector Analysis Versus Vector Calculus, Springer, London, 2012.
- [5] Karger, A., Novak, J., Space Kinematics and Lie Groups, Gordon and Breach Science Publishers, 1978
- [6] O'Neill, B., Semi-Riemann Geometry: with Applications to Relativity, Academic Pres, New York, 1983.
- [7] Uğurlu, H.H., Kocayiğit, H., *The Frenet and Darboux Instantaneous Rotation Vectors of Curves on Time-like Surface*, Mathematical & Computational Applications, 1 (1996) 133–141.
- [8] Yaylacı, T., *Linear Vector Fields and Applications*, MSc thesis, Ankara University, The Institute of Science, Ankara, 2006.

SECTION 5

Probability, Statistics, Numerical Mathematics and Optimization

Using Informative Prior from Meta Analysis in Bayesian Approach

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Bayesian approach differ from the frequentist approach in terms all unknown parameters are considered as random variables. For that, prior distribution must be defined at first. Determination of the prior distribution is important in because it effect the posterior inference. If prior information is available, it should be appropriately summarized by the prior distribution. Such distributions are called informative distribution. In the case of no prior information is available, we define prior as a will not effect the posterior distribution [1].

Meta analysis refers to the statistical synthesis of results from a series of studies. If the studies have been collected systematically, the synthesis will be meaningful. Provide more powerfull test, summerise numerous and inconsistent findings and investigate consistency of effect across different samples are the reasons of using meta analysis [2].

The objective of this study is to used meta analysis for proportion to obtain prior information about patients with breast cancer stage-I who received modified radical mastectomy treatment and applied Bayesian approach. R and WinBUGS programs are performed for meta analysis and Bayesian approach respectively.

- [1] I. Ntzoufras, *Bayesian Modeling Using WinBUGS*, Hoboken-New Jersey-USA, Wily & Sons, 2009.
- [2] M. Borenstein, L.V. Hedges, J.P.T. Higgins, H.R. Rothstein, *Introduction to Meta-Analysis*, West Sussex-UK Wily & Sons, 2009.

Regularized least-square optimization method for variable selection in regression models

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A new type of regularization in least-square optimization for variable selection in regression models is proposed. Proposed regularization is suitable for regression models with equal regressors' influence. Consistency of the estimator of the regression parameter under suitable assumptions is shown. Numerical results demonstrate efficiency of the proposed regularization and its better performance compared to existing regularizations.

Clinical evaluation of treatment of gingival recessions supported by Bayesian statistics

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Gingival recession is a common manifestation in most populations. Gingival recession may be a concern for patients for a number of reasons such as root hypersensitivity, erosion, root caries, and esthetics. Multiple gingival recessions may be a concern for patients with a high lip smile line. Studies on this surgical challenge mostly concern the treatment of recession defects. Multiple adjacent recession-type defects present a further challenge because several recessions must be treated at a single surgical session to minimize patient discomfort.

VISTA (vestibular incision subperiosteal tunnel access) is a novel, minimally invasive approach applicable for both isolated recession defects as well as multiple contiguous defects in the maxillary anterior region. A 6-month postoperative measurement period is sufficient to evaluate the stability of the gingival margin.

The aim of this study is to offer an approach for comparing sample results with the clinical outcomes reported in several case studies.

We will compare our clinical results in terms of root coverage with the clinical outcome reported in a case study.

We will do practical analysis with a representative sample. Markov chain Monte Carlo (MCMC) algorithms and software, along with fast computer hardware, allows us to do this Bayesian data analysis ([2,3]). In the case study reported in [1] gingival recession was measured at 6 month post-surgery and root coverage of 91% was obtained. The question is whether the value of interest falls among the most credible values in the posterior.

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- [1] Chatterjee A., Sharma E., Gundanavar G., Subbaiah SK, *Treatment of multiple gingival recessions with vista technique*, A case series, J Indian Soc Periodontol, 19(2) (2015), 232235.
- [2] Gelman, A., Carlin, J. B., Stern, H. S., Rubin, D. B. *Bayesian data analysis*, Boca Raton, Chapman and HallCRC, Third Edition, 2013
- [3] Kruschke, J. K., *Doing Bayesian data analysis: A tutorial with R and BUGS*, Burlington, MA: Academic Press/Elsevier, Second Edition, 2011

An example on application of randomly indexed central order statistics

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We recall the asymptotic behavior of randomly indexed upper order statistics using regular norming time-space changes, define the model for a point process $\mathcal{N} = \{(T_k, X_k) : k \geq 1\}$ and give an example of application of the transformed model under additional assumptions.

Using Queueing theory-M/M/1 in Hospital Tetovo

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Modern information technologies require innovations that are based on modeling, analysing, designing and finally implementing new systems. The whole developing process assumes a well-organized team work of experts including engineers, computer scientists, mathematicians, physicist just to mention some of them. Modern infocommunication networks are one of the most complex systems where the reliability and efficiency of the components play a very important role. The aim of the paper is to present a basic method, approaches in a Markovian level for the analysis of not too complicated systems by using M/M/1 Queueing systems. Ours experience and advice are that if it is possible solve the same problem in different ways and compare the results. In this paper the concrete problem is expected in the laboratory blood tests at the hospital in Tetovo (Republic of Macedonia). Basis for achieving results was done an analysis on basis of a questionnaire and is studying the phenomenon for three months. From the data provided is concluded that on average 120 patient counter attainment of services on that day according to the model M / M / 1 represents Poisson's membership ians with parameter λ (arrival/time). But at the time of service pose an exponential ians μ (patient/time) of two parameters.

Stochastic approximation with adaptive step sizes for optimization in noisy environment and its application in regression models

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We propose a method for optimization problems in noisy environment. The method represents a modification of Stochastic Approximation algorithm based on a new adaptive step sizes scheme and it is extended to case where general descent direction is taken instead of noisy gradient. Proposed adaptive step size scheme uses only a predefined number of last noisy functional values to select a step size for the next iterate. The almost sure convergence is established under suitable assumptions. Numerical results indicate a good performance of the method. Application of the method in regression models is presented.

Complex-step gradient approximation and performance analysis of nonmonotone optimization algorithms in noisy environment

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The complex-step derivative approximation is powerful method for derivative approximation which has been successfully implemented in deterministic numerical algorithms. We explore its implementation in noisy environment through performance analysis of nonmonotone line-search optimization algorithms. Numerical results show good performance of the complex-step gradient approximation in noisy environment.

Imputation of missing categorical values in survey research data

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The problem of bias due to missing data has received a good deal of attention over the last 20 years and the correction of bias due to non-response remains an important problem for investigators using survey data. For data missing because of item nonresponse, imputation of the missing data is often the best solution. Many software packages will automatically remove cases with missing values from the analysis, greatly reducing the sample size, often causing a drastic loss of information. Additionally, if the data are not missing completely at random, removing cases with missing items will result in biased parameter estimates in subsequent analyses.

This study discusses a new method for imputation of missing survey data with a large number of categorical variables. Our method for direct ascription of missing categorical values needs both a tool for association and a tool for detecting which parts of the table are responsible for this association. We use the ubiquitous chi-square test for association in a cross-tabulation. However this test is not a tool detecting which parts of the contingency table are responsible for this association. Correspondence Analysis (see [1], [2]) is a tool that can fill this gap, allowing the machine learning algorithm to see the pattern of association in the data and to generate hypotheses for ascription of missing values that can be tested.

This method for direct ascription of missing categorical values is based on both the association between row and column points and the inertia.

The result can be visualized by plots. The graphical solution is restricted to two dimensions. A three-dimensional display can also be created. This type of display offers the advantage that one can zoom and navigate using the mouse.

We report the result of imputing the missing data from a survey dataset.

Acknowledgments. The research is supported by the Fund NPD, Plovdiv University Paisii Hilendarski, under Grant NI15-FMIIT-004.

- [1] Greenacre, M., Correspondence Analysis in Practice, Second Edition. London: Chapman & Hall / CRC, 2007
- [2] Nenadic, O., Greenacre, M., Correspondence Analysis in R, with Two- and Three-dimensional Graphics: The ca Package, Journal of Statistical Software; Vol. 20, No. 3, 2007

A Game Theoretical Approach for Supplier Interdiction on Single Echelon Supply Chain in Competitive Environment

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Game theory is used widely in many scopes in the literature. In this talk, considering network interdiction approach whose main logic is based on game theory, we analyze the competitive procurement process of two firms which are called leader and follower. It is assumed that these firms, which have the same activity area, operate in the same region and use the same supply chain. It is naturally expected that the firms need same materials (i.e. suppliers). This may put the production levels of each other in danger as the amount of materials in suppliers will reduce. This risk compel the firms to seek various policies for surviving in this competitive environment. The main goal of this talk is to help decision maker(s) (leader) for determining their interdiction policies that will damage to the rival firm (follower) and also increase the leaders profit under changing competitive conditions. This process is modeled as a bi-level mixed integer program and is converted into mixed integer program where the leader explicitly maximizes the minimum total cost (achieved by follower) with interdicting the supplier(s). Finally, significant approaches that help the leader to determine the interdiction plans related to his goals are presented on an explanatory numerical study.

An optimal set of quadrature rules for trigonometric polynomials

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In this paper we consider an optimal set of quadrature rules with an odd number of nodes for trigonometric polynomials in the sense of Borges [Numer. Math. **67** (1994), 271–288]. As a matter of fact we consider evaluation of a set of $p \in \mathbb{N}$ definite integrals related to a common integrand over the same interval E of length 2π , but taken with respect to the different weight functions. The optimal set of quadrature rules is characterized by multiple orthogonal trigonometric polynomials of semi–integer degree. We give main properties of such multiple orthogonal trigonometric system as well as the numerical procedure for constructing the corresponding quadrature rules. Theoretical results are illustrated by some numerical examples.

Identification of a fuzzy measure that meets redundancy among criteria in multicriteria decision making problems

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Fuzzy measure is an innovate and useful tool to model the interaction of criteria in a multicriteria decision making (MCDM) environment. However, to determine the weight of each one of 2^n subsets of a finite set that has cardinality n is complicated. Larbani et al. relieve this complexity by constructing a fuzzy measure which has been developed based on the evaluation of interdependence coefficients between criteria. On the other hand the structure of the set function has been referring the superadditivity among criteria and fortunately the construction of the function is convenient with the structure of superadditive fuzzy measures. In contrast to this case it is not an easy issue to obtain a fuzzy measure that refers subadditivity between criteria by using a set function with similar structure. As it is necessary to use a negative interaction index for each pair of criteria to ensure redundancy between singletons; keeping the monotonocity and subadditivity is more complicated. In this talk we are concerned with this crucial problem. We present independent necessary and sufficient conditions to attain a fuzzy measure from this set function. Later, we discuss these conditions and we support the results with explanatory numerical examples.

SECTION 6

Actuarial Mathematics, Mathematical Finance and Modeling in Economy

An approach to the income function movement when its components change according to the principle of the geometric progression

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The income as a component plays a great role in the overall operation of an enterprise. Its variations are directly related to survival and principles of liquidity and profitability of the operation of a company. Although in theory, in the process of mathematical modeling, the income is defined as monotonous growing function, in practice it turns out that deviations from such expectations are greater. Analyzing the factors that affect the movement of income, we can predict its future trends and with it, we can react appropriately to prevent large losses for the company. Reduction of the costs as a key component of the income function, intuitively leads to the conclusion that the profit will grow, during this presentation through a concrete example in the analysis of the reports of a company with greater relevance for the Macedonian economy, we will show that cost reduction not always leads to profit increase. In fact we will try in some way to determine the movements of the income curve in cases, when the costs are moving as a geometric progression, and thereby optimize, maximize revenue, directly or indirectly reacting on the other components of the function of income.

SECTION 7

Mathematical Aspects of Computer Sciences

Mathematical calibration of sensors for embedded applications

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Mathematical aspects of the computer science are fundamental in the real-life implementations. One of these is the computer interaction with the real world. Modern trends in computerized automation involve use of sensorics and actuation, and computers (embedded systems) need to acquire physical phenomena in order to decide/calculate response. Crucial aspect of data acquisition is calibration of sensory output in order to comprehend the real physical value. This case study depicts the process of proper interfacing a computer with a NTC thermal probe in order to read current temperature of the environment the probe measures. The process begins with designing a basic electronics for required current excitation of the NTC probe. Than voltage it produces over its resistor is acquired via an A/D converter, it is sampled and digitized. It is calibrated for the preamplification and finally a set of exponential equations are used to determine key parameters for voltage calibration to convert it into temperature. Than decisions can be made consistent to the embedded application - home automation in this particular case study. Data used for modeling of the resistance-temperature dependence is issued by the probes manufacturer, and equations are already well known physical relations.

A pi-calculus ADL for semi-automatic code generation

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Software architectures are used to describe the structure of the components of a program/system, their interrelationships, and principles and guidelines governing their design and evolution over time. The runtime behaviour can be presented by the component-and-connector architectural view where components represent the primary computational units and connectors represent the communication between components. In order for software architectures to be expressed (design, evaluation etc.) formal description languages are used, called Architecture Description Languages (ADL). Although ADLs could be proved to be a very powerful tool in the hands of architects, for a number of reasons (e.g. too formal language constructs), in practice they are not used by software developers in development process these days.

In this paper we present a new ADL, named jADL, which targets on automatically generating implementation code stubs providing a software tool not just for software architects but for developers as well, that ensures the consistency between the architecture and the actual implementation. jADL is a well-formed extension of a process calculus developed for mobile processes typed polyadic asynchronous pi-calculus. The pi-calculus is a process algebra for communicating processes with a dynamically changing topology which have been used for specification of concrete software architecture topology and behavior during run-time of a software system.

In this paper we briefly will present the principles and notation of jADL through simple examples. We will present pi-calculus extensions and we will establish strong type system useful for code generation.

Supporting education in math sciences with a low-budget LMS SAKAI

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Informational-communicational technologies nowadays are big trend in modern lifestyle, without which working and living is unimaginable. They help saving money, time and other resources by using different forms of computerized services. In this presentation the accent will be put on the Learning management system SAKAI and its application in supporting education in math sciences. SAKAI as low-budget LMS is free and offers a lot of opportunities in the educational process. SAKAI provides comprehensive content for each course, like: forums, announcements, materials, exam results, event calendar, personalization posibilities etc. The plugins offered by SAKAI allow powerful presentation capabilities for mathematical courses (formulas, equations, graphs etc.), one of them being the WIRIS. The WIRIS plugin is the component needed present in a web application enabling the use of the WIRIS editor - a WYSIWYG JavaScript based equation editor that provides seamless integration with any application including on-line communication in support to instant formulae representation and exchange. A WIRIS plugin interacts with the platform by a whole set of plugins targeting different technologies (PHP, Java, ASP .NET, etc.) and different HTML editors (TinyMCE, CKEditor, etc.) or available platforms (Moodle, Canvas, SAKAI, Joomla, etc.). The presentation will be focused on the SAKAI implementation.

SECTION 8

History of Mathematics and Mathematics Education

Online homework systems in Calculus

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In this presentation we will discuss the effectiveness of online homework in Calculus courses. Specifically, we will discuss the use of WeBWorK – an open-source online homework system supported by the Mathematical Association of America and the National Science Foundation. We will confer practices for implementing WeBWorK homework for Calculus and discuss student learning using this online homework system.

Teaching online mathematics course for undergraduate students

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Distance learners are growing in numbers and universities respond to this growth by offering online courses. Students like online courses and distance degree programs for their flexibility, ease of access and affordability. Many educators are looking for training or workshops on how to improve their online teaching skills. In this presentation I will discuss what I learned about online mathematics courses, share my experience of teaching an online course, and what I would suggest for those about to teach online.

Mathematical terminology: On the names of fractions in the Macedonian language

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Mathematicians use mathematical language to communicate mathematical ideas and results among themselves. This language consists of some natural language (for example Macedonian) with the usual grammatical conventions and mathematical terms, supplemented by a highly specialized symbolic notation for mathematical formulas. We discuss the question of the names of fractions in the Macedonian language and explain our position in using consistent system for generating nouns that denotes fractions, a system that is unambiguous and clear, unique and complete.

Names of foreign mathematicians in the mathematical literature in Macedonian

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In this talk we discuss the problem of pronunciation and transcription of the names of foreign mathematicians in the teaching of college mathematics and in mathematical literature in Macedonian. We adapt to the language rules and the transliteration challenges of the original names and state some examples from different textbooks and from the practice, which require pronunciation and transcription improvement.

An overview on the curricula for linear algebra in the Macedonian gymnasiums

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In this talk, I would discuss the current state of the curriculum for Linear algebra in the upper high schools, called gymnasiums with a profile for natural sciences and mathematics in the Republic of Macedonia. The discussion would consider two aspects. First, I would compare the old with the actual version of the curriculum dating from 2014. Second, I would compare the current curriculum with appropriate curricula in the west European cultures as Germany and France. These comparisons will point out not only the selection of the content-specific topics based on an epistemological analysis but will also include didactic considerations and suggestions for teaching and learning. The argumentation is based on research evidence related to the teaching and learning of Linear algebra through lower secondary, upper secondary and tertiary level of education with an emphasis on the transition periods. Further on, I would suggest Situations, according to the Theory of Didactic Situations (TDS) (Brousseau, 1986), for innovative teaching of Linear algebra. In conclusion, the overview could offer proposals for further researchbased curriculum development which could take into account the suggested innovations. The talk might attract the attention of textbooks authors besides curricula designers and teachers.

Better integrated teaching of geometry and algebra in primary education

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The integrated teaching of mathematics is important for the development of the students from every aspect. Practice shows that integrated knowledge improves the quality of thinking and at the same time enables the students to acquire permanent and applicable knowledge. Although there are declarative efforts for acquiring integrated knowledge, there is lack of inter-subject and intra-subject integration. This paper scrutinizes how elementary geometry knowledge can be used to improve integrated teaching. We are going to show how to come up with formulas for some sums of numbers by using areas of squares and rectangles. The latter is especially important since these formulas are presented to the students either in a dogmatic way, or through mathematic induction, which is incomprehensible for most students of this age.

Logical fallacies as a tool for empowering students' thinking and reasoning in math classes

VALENTINA GOGOVSKA

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Development of thinking and in particular the development of mental qualities – width, depth, independence, logic, mobility, concreteness, criticism, speed, creativity, target orientation, generalization, insight, etc. is one of the most important and consistent goals and objectives of the math teaching.

Simultaneously, the degree to which this aim is fulfilled determines the level and effectiveness of the teaching process for the overall development of the students' personality. An important psychological and pedagogical condition for the development of quality of thinking is students' reflexive understanding of thinking as a process and their own mental capabilities.

This work attempts to promote the use of logical fallacies during every day mathematics classes, especially during classes for exercises trough a few examples. Well-chosen examples can improve and empowered the process of doing mathematics, and can stimulate the process of creative thinking and motivate students' individual development in their current learning and leads to the formation of intellectual reflection.

Computer discovered mathematics and applications in education

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Since the beginning of the computer era the discovery of new mathematics by computers is an important research task. In 1958, in a seminal paper predicting future successes of artificial intelligence, Herbert Simon and Alan Newell suggested that: Within ten years a digital computer will discover an important new mathematical theorem. Now is year 2015, and computer discovered theorems are already published. The first computer program able easily to discover new theorems is the "Discoverer", the computer program created by Grozdev and Dekov. Hence, now we can tick off the prediction by Simon and Newell as realized.

In the first part of the talk we discuss the Paulson criterion which determines the data necessary to accept the Simon-Newell prediction as realized. In the second part we present a survey on mathematical results discovered by the "Discoverer".

We consider a computer-discoverer as effective, if it is an every-day tool for investigation by mathematicians. In the third part of the talk we present applications of computer discovered results in the mathematics education. We accent on the production of problems for high-school and university education by the "Discoverer".

Early learning of mathematics and the use of picture books in teaching

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The research study reported in the paper is focused on exploring the effects of picture book reading in grade 1 mathematics classrooms. The goal is to understand the power of stories and of illustrations in picture books to stimulate in children cognitive processes which encompass the formation of mathematical concepts and the development of mathematical thinking. The underlying theoretical perspectives are based on a (social) constructivist approach to learning grounded in the work of Piaget and that of Vygotsky. A description of the process of selection of picture books based on characteristics which are postulated to support mathematics learning is given, as well as an account of videotaped sessions with small groups of 6-year old pupils being read a selected picture book in some of which the teacher used appropriately chosen questions to enhance pupils reactions. Observed verbal reactions are recorded based on the written transcripts and then categorized using a framework of mathematical reactions. Correlations between pupils level of cognitive engagement measured via the frequencies of their mathematical reactions during the picture book reading and their achievements on a specially prepared test administered at the end of the first semester are discussed.

Age factor in a math contest paper for 5th and 6th grade students

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The format of the Chernorizec Hrabar Math Tournament includes 5 divisions for students of two consecutive grades, e.g. 3rd and 4th or 5th and 6th grade students. Natural question is how appropriate is the common competition paper for any of two age strata. To clarify such issue we introduce the notion of age-factor index as the difference of the test-item difficulties calculated separately for any age strata, e.g. $\Delta = T_5 - T_6$ where T_5 and T_6 stand for the test-item difficulty for the 5th and 6th grade students' variation row respectively. The average value of Δ for the contest paper could be considered as a relative measure of reliability and uniformity of the contest paper for a particular target group. The study presents details about 2015 Chernorizec Hrabar contest paper for 5th and 6th graders and the application of the new concept in didactical analysis.

Geometrical solving of some systems of equations

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We present a geometrical way for solving some systems of trigonometric, exponential, logarithmic and irrational equations. Equations are transformed and solved geometrically.

Examination of the students' previous knowledge before and after the first year of secondary education

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The previous knowledge of the students enroled in the first year in the "Georgi Dimitrov" Skopje high school was examined in the 2013/2014 and 2014/2015 school year. The students enroled came from 15 primary schools in Skopje. We believe that this examination is a good indicator of the quality of the knowledge necessary for understanding the math curriculum in the first year of high school. A similar examination was made at the beginning of the mentioned school years to a group of students who enroled the second year of high school and on the students who enroled the third year of high school at natural sciences and mathematics A level.

Most frequent errors and misconceptions of first year math students in high school problems

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A four-year study that includes first year math students at PMF, UKIM - Skopje, is made. The students got an anonymous test containing six different assignments of logical type, linear equations with parameter and systems of two linear equations with two variables containing parameters. We consider their results from the application of prior math knowledge at the test, we classify the most common errors when solving equations and systems of equations with parameters, and look into the misconceptions that arise when adopting the notions of ordering relations.

Application of math knowledge in solving problems of geometrical optics

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When solving problems of geometrical optics, besides physics, it is inevitable that students use the standard geometrical apparatus. Even when the basics of physics are well known, the possible mathematical mistakes often lead to misinterpretation of the results in physical sense. Here we demonstrate an example of such misinterpretation on selected problems for Macedonian state competitions in physics. The sample of students considered are primary and high school students involved in physics competitions in the year 2016.

Algebraic strategies for predictions in elementary mathematics education

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Algebraic thinking and reasoning is the process in which we generalize mathematical ideas from a specific set of instances and the conclusion of the generalization is represented in a different way suitable to students' age. The importance of algebraic thinking is a challenge to implement it and to demonstrate it in practical examples. We have chosen the patterns as an initial step in boosting and developing the skills of algebraic and mathematical thinking among students in general. In this regard, we will present several examples of different types of patterns of shapes and numbers and ideas about their treatment by elementary school students. If you create a working atmosphere in which students are curiously involved, they can reveal the rules that lie under the patterns, they can classify the patterns in order to continue the pattern and to generalize the rule. Step above the generalization is the detection of the functional dependence.

Infinity in mathematics teaching in primary school

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In this presentation we discuss the concepts of geometry and arithmetic that are introduced in the initial teaching of mathematics and which contain the idea of infinity. For young pupils (as well as for the ancient Greeks) infinity is primarily potential infinity which has solid support in their intuition.

Analysis of tasks and tests in mathematics for eighth grade in nine-years primary school

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The objective of teaching any subject, including mathematics is an active and creative proficiency of knowledge that will be durable and applicable in practice. Therefore, in addition to the appropriate educational contents, the relevant modern teaching tools, the adequate school supplies and the motivation among the parents and the students, it is necessary to have competent teachers, as the principal means of any national school system.

A very important segment of the teachers' education is their ability for preparing and evaluating the tests for knowledge of particular subjects independently. The question is whether in the teaching practice there is someone who is competent enough of producing tests of knowledge, satisfying all metric features which turn the test into a measuring instrument of the pupils' knowledge?

The subject of this paper is to determine to what extent the teachers at mathematics in primary schools are able to prepare tests of knowledge and to define their metric characteristics. Moreover, in this paper we determine the metric characteristics specifically of the tests of knowledge at mathematics in the eighth grade from nine-year primary schools.

Mathematical modeling in mathematics education through solving realistic Fermi problems

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Mathematical modeling is getting more and more attention in mathematics education in recent years. The use of mathematical modeling in mathematics education makes the problem solving process to be more approachable for students, students become more interested in it, they learn to make decisions based on detailed reasoning, they learn to explore. One type of problems that can be used for mathematical modeling are Fermi problems, which are open, non-standard problems, requiring an estimate of some quantity. Calculations that are needed, may not be demanding, but logic, critical thinking, extra-mathematical knowledge, as well as the skill for dividing more complex problems into smaller, solvable parts, are needed. We will present the results from the research conducted at Educational Centre "Abakus Nib - Ohrid" in Ohrid, where the work of four groups of high school students in third year and their teacher on solving Fermi problems has been observed. Quantitative and qualitative analysis of collected data have been done. The process of mathematical modeling during solving Fermi problems is discussed, together with the interaction among the members in a group.

Solving real-life problems for increasing the motivation of students in learning mathematics

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One of the priorities of the European education and in our country in general is increasing the knowledge of students in Mathematics. Students use computers, mobile phones and other devices to satisfy their everyday needs. If we manage to encourage the students and motivate them to use these electronic devices for testing, research and solving problems which they encounter in everyday situations, they can be motivated to learn mathematics and at the same time their level of knowledge will be increased.

For this purpose there is an interactive book Math Labyrinth developed on a web site math-labyrinth.eu.

The problems uploaded on the web site are in accordance with the contents and topics covered in the curricula for secondary education and in relation with everyday situations. These problems require knowledge in mathematics in order to be solved. After reading and understanding the problem, if the student knows the procedure for solving, he/she will begin doing the task. They will also be able to check whether the answers are identical to the given solution. If the students experience some difficulties they can look at the provided aplets that enable visualization of the problems and then choose path with steps and hints. By going through the given path that explains the procedure and using ICT, the student can come to the solution by himself.

POSTER SESSION

Statistical approximation properties of bivariate extension of q-Lupa-Stancu operators

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In this paper we introduce a bivariate generalization of the q-Lupa-Stancu operators. First, we estimate the moments for these operators, then we examine approximation properties of our new operator via Korovkin type theorem. We also establish approximation properties and rate of A-statistical convergence for these operators. Further, the error of approximation for these opperators is calculated by using modulus of contuinuity and Lipschitz-type functions. Finally, the convergence of the bivariate q-Lupa-Stancu operators for certain functions are illustrated graphically using Python programming language.

Modified AES for increasing data security and its parallel implementation

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Secure communication has been longstanding concern of mankind, starting by the Code of Caesar and until today algorithms with symmetric, asymmetric encryption and hash functions to ensure data integrity. We have built an MDS matrix (Maximal Separable Distance) that will be added during the AES algorithm encryption or decryption process. We have modified the AES algorithm to boost its security, which in fact has increased the number of calculations within the algorithm rounds. That the greater number of calculations have no impact on coding or decoding delay at the end we propose the parallel version of this algorithm.

Math teachers' past and current experience affecting their future test grading

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This poster presents the results obtained in a research, the idea for which was born during my post-graduate studies. The research was based on three questions. The first one was aimed at learning the difference in the grading of 5 separate math problems without a previously given key, with an assigned maximum number of points. The second question deals with the relation between the teachers' grading and the years of their experience. The last question deals with the difference between the grading of teachers teaching in a secondary school compared with the ones of teachers working in a primary school. What was analyzed were 5 tests graded by 50 math teachers from various cities in the Republic of Macedonia, teaching in primary and secondary schools. The results of this research might be especially useful for raising the awareness of a fair and objective assessment and the ways it can be achieved by taking a critical look at the way we grade our students.

WORKSHOP 1

International Workshop on Generalized Functions and Pseudo-differential Operators

Workshop Description:

The main purpose of the workshop is to promote modern research developments on the interaction of the fields of generalized functions and pseudo-differential operators, as well as their applications to other areas of mathematics. The workshop has an international character; it will bring together national and international experts on these two subjects, young researchers, and doctoral students. It is organized within the framework of the VI Congress of Mathematicians of Macedonia (Ohrid, June 15-18, 2016).

Topics include (but are not limited to):

- 1. Distribution and ultradistribution theory, hyperfunctions, boundary values, and integral transforms;
- 2. Pseudodifferential operators, microlocal analysis, Fourier integral operators, and spectral theory;
- 3. Harmonic analysis and time-frequency analysis;
- 4. Applications, including differential equations, asymptotic analysis, and analytic number theory.

Duration: 3 days.

Co-organizers:

- Stevan Pilipović, Department of Mathematics and Informatics, Faculty of Sciences, University of Novi Sad, Serbia
- Bojan Prangoski, Faculty of Mechanical Engineering,
 Ss. Cyril and Methodius University, Skopje, R. Macedonia
- Jasson Vindas, Department of Mathematics, Ghent University, Belgium

Workshop leaders:

- Bojan Prangoski (bprangoski@yahoo.com)
- Jasson Vindas (jvindas@cage.ugent.be).

Participants (in alphabetical order):

- Sanja Atanasova, Ss. Cyril and Methodius University, R. Macedonia
- 2. Marco Cappiello, University of Turin, Italy
- 3. Sandro Coriasco, University of Turin, Italy
- 4. Andreas Debrouwere, Ghent University, Belgium
- 5. Gregory Debruyne, Ghent University, Belgium
- Pavel Dimovski, Ss. Cyril and Methodius University, R. Macedonia
- 7. Smiljana Jaksić, University of Belgrade, Serbia
- 8. Dennis Nemzer, California State University, Stanislaus, USA
- Daniel Lorenz, Technical University Bergakademie Freiberg, Germany
- Alessandro Palmieri, Technical University Bergakademie Freiberg, Germany
- 11. Stevan Pilipović, University of Novi Sad, Serbia
- 12. Bojan Prangoski, Ss. Cyril and Methodius University, R. Macedonia
- 13. Maximilian Reich, Technical University Bergakademie Freiberg, Germany
- 14. Michael Reissig, Technical University Bergakademie Freiberg, Germany
- 15. Luigi Rodino, University of Turin, Italy
- Katerina Saneva, Ss. Cyril and Methodius University,
 R. Macedonia
- 17. Petar Sokoloski, Ss. Cyril and Methodius University, R. Macedonia
- 18. Joachim Toft, Linnaeus University, Sweden
- 19. Filip Tomić, University of Novi Sad, Serbia
- 20. Ville Turunen, Aalto University, Finland
- 21. Daniel Velinov, Ss. Cyril and Methodius University, R. Macedonia
- 22. Jasmina Veta Buralieva, "Goce Delchev" University, R. Macedonia
- 23. Jasson Vindas, Ghent University, Belgium
- 24. Ivana Vojnović, University of Novi Sad, Serbia
- 25. Đorđe Vucković, Ghent University, Belgium

H - distributions with unbounded multipliers

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H-measures were introduced independently by Tartar, [4] and Gérard, [3]. They are used to determine weather a weakly convergent sequence in $L^2(\mathbb{R}^d)$ converges strongly. Antonić and Mitrović in [2] introduced H - distributions, extension of H - measures to an $L^p - L^q$ setting for 1 and <math>q = p/p - 1, . In [1], H - distributions are constructed for sequences in dual Sobolev spaces, $W^{-k,p} - W^{k,q}$. Test functions for H - measures and H -distributions are bounded Fourier multipliers.

We construct H - distributions for sequences in dual Bessel potential spaces, $H_q^k - H_p^{-k}$ and in this case test function can be unbounded multiplier. Also a necessary and sufficient condition is given so that the weak convergence in H_p^{-k} implies the strong one. Results are applied on a weakly convergent sequence of solutions to a family of pseudo-differential equations.

REFERENCES

- [1] Aleksić, J.; Pilipović, S.; Vojnović, I. *H distributions via Sobolev spaces*, Mediterranean Journal of Mathematics, 2016., pp 1-14
- [2] Antonić, N.; Mitrović, D. *H-distributions: an extension of H-measures to an* $L^p L^q$ *setting.* Abstr. Appl. Anal. 2011, Art. ID 901084, 12 pp.
- [3] Gérard, P. Microlocal defect measures. Comm. Partial Differential Equations 16 (1991), no. 11, 1761–1794.
- [4] Tartar, L. H-measures, a new approach for studying homogenisation, oscillations and concentration effects in partial differential equations. Proc. Roy. Soc. Edinburgh Sect. A 115 (1990), no. 3-4, 193–230.

Asymptotic behavior of distributions and time-frequency analysis

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The idea of this talk is to make sublimation of the joint work of the authors in the last few years. The focus will be on the analysis of the asymptotic behavior of distributions in terms of different integral transforms. The short-time Fourier transform, directional STFT and Radon transform will be discussed in various test function and distribution spaces, and new kind of Abelian and Tauberian results connected to them will be presented. Also, the interest of this talk will be characterizations of asymptotic properties of Schwartz distributions by using Gabor frames.

Pseudodifferential operators and wave front sets on quasi-analytic Gelfand-Shilov spaces

MARCO CAPPIELLO,

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The talk is focused on the challenging problem of developing a global pseudo-differential theory and to define wave front sets on quasi-analytic function and distribution spaces. The lack of compactly supported functions in this setting makes this analysis rather difficult. On the other hand, recent results show that some interesting operators in Mathematical Physics satisfy quasi-analytic estimates, see [1]. We first consider some classes of symbols related to the harmonic oscillator which motivate this analysis, and we discuss the possibility of constructing a global calculus in these classes. Moreover, we derive some regularity and decay estimates for PDE involving these operators. Finally we introduce global wave front sets for tempered ultradistributions and investigate their applications. The proofs of some of these results are based on a combination of standard microlocal techniques with tools coming from time-frequency analysis and they have been obtained in collaboration with L. Rodino, R. Schulz and J. Toft.

- [1] M. Cappiello, L. Rodino, J. Toft, *On the inverse to the harmonic oscillator*, Comm. Partial Differential Equations **40** (2015) no. 6, 1096-1118.
- [2] M. Cappiello, R. Schulz, Microlocal analysis of quasianalytic Gelfand-Shilov type ultradistributions, Complex Var. Elliptic Equ. 61 (2016) no. 4, 538-561.
- [3] M. Cappiello, J. Toft, Pseudo-differential operators in a Gelfand-Shilov setting, Math. Nachr. (2016). To appear.

A sharp Weyl formula for tensor products of pseudodifferential operators

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I will illustrate results concerning the asymptotic behavior of the counting function of tensor products of operators, in the cases where the factors are either pseudodifferential operators on closed manifolds, or pseudodifferential operators of Shubin type on \mathbb{R}^n , respectively. In particular, I will discuss the sharpness of the remainder term in the corresponding Weyl formulae. This is joint work with U. Battisti and M. Borsero.

REFERENCES

[1] Sharp Weyl Estimates for Tensor Products of Pseudodifferential Operators. *Ann. Mat. Pura Appl.* DOI 10.1007/s10231-015-0490-2 (2015).

Non-triviality of certain spaces of analytic functions. Ultrahyperfunctions and hyperfunctions of fast growth.

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The test function spaces for Fourier ultrahyperfunctions and Fourier hyperfunctions (in one dimension) consist of functions φ which are analytic on the horizontal strip $|\operatorname{Im} z| < k$ and satisfy

$$\sup_{|\operatorname{Im} z| < k} |\varphi(z)| e^{k|z|} < \infty,$$

for each k>0 and some k>0, respectively. In this talk we are interested in the following generalization of these spaces: let M be a non-decreasing positive function defined on the positive half-axis satisfying certain natural conditions – e.g. the associated function of a weight sequence M_p – and consider the spaces consisting of functions φ which are analytic on the strip $|\operatorname{Im} z| < k$ and satisfy

$$\sup_{|\operatorname{Im} z| < k} |\varphi(z)| e^{M(k|z|)} < \infty,$$

for each k>0 and some k>0, respectively. Our results are twofold. Firstly, we present an analytic representation theory for the duals of these spaces and express them as cohomological quotients of spaces of analytic functions. Secondly, by using the aforementioned results, we characterize the non-triviality of these test function spaces in terms of the growth of the weight function M. In particular, we show that the Gelfand-Shilov spaces of Beurling type $\mathcal{S}^{\{p!\}}_{\{M_p\}}$ and Roumieu type $\mathcal{S}^{\{p!\}}_{\{M_p\}}$ are non-trivial if and only if

$$\sup_{p>2}\frac{(\log p)^p}{h^pM_p}<\infty,$$

for all h > 0 and some h > 0, respectively.

Complex Tauberian theorems for Laplace transforms

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Tauberian theorems have proven to be useful in diverse areas of mathematics such as number theory and spectral theory of differential operators. The Tauberian theorems that we will discuss will be of the following type. Under some regularity assumptions on τ the asymptotic relation $\tau(x) = o(1)$ can be deduced from certain boundary behavior of the Laplace transform $\mathcal{L}\{\tau;s\}$ on the line $\Re e\, s = 0$. The regularity assumptions are usually called Tauberian conditions and are typically boundedness or monotonicity conditions. For many Tauberian conditions, local pseudofunction behavior of $\mathcal{L}\{\tau;s\}$ on the line $\Re e\, s = 0$ has been shown to be necessary and sufficient to deduce $\tau(x) = o(1)$. We will present some of the latest developments in this field. The talk is based on collaborative work with Jasson Vindas.

Modulation spaces related to translation-invariant Banach spaces of quasi-analytic ultradistributions

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We define and study a new class of translation-modulation invariant Banach spaces of quasi-analytic ultradistributions. These new spaces show a certain stability under Fourier transform, duality and tensor product. Multiplication of the Fourier Lebesque spaces $L^1_{\hat{\omega}}$ with elements from these spaces, also multiplication of elements from this space with elements from its dual are considered. We associate a new Banach space \mathcal{M}^F to translation-modulation invariant Banach space F. These space \mathcal{M}^F remains translation-modulation invariant Banach space. The duals of \mathcal{M}^F are also considered. The new defined spaces \mathcal{M}^F and results concerning them are generalizations of already known Modulation spaces of (ultra)distributions and results about them.

Structure theorems for G-type spaces of ultradistributions over \mathbb{R}^d_+

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Two structure theorems for the *G*-type spaces of ultradistributions over $[0, \infty)^d$, i.e. $(G^{\alpha}_{\alpha}(\mathbb{R}^d_+))'$, $\alpha \geq 1$ are given. The first one states that $f \in (G^{\alpha}_{\alpha}(\mathbb{R}^d_+))'$, $\alpha \geq 1$ if and only if it can be written as

$$f = \left(\sum_{k \in \mathbb{N}_0^d} c_k \left(x D^2 + D - \frac{x}{4} + \frac{1}{2} \right)^k \right) F,$$

where the coefficients c_k have a suitable growth and $F \in L^2(\mathbb{R}^d_+)$. The second one uses the fact that $G^{\alpha}_{\alpha}(\mathbb{R}^d_+)$, $\alpha \geq 1$ is given as an injective inductive limit of Fréchet spaces and loosely speaking represents $f \in (G^{\alpha}_{\alpha}(\mathbb{R}^d_+))'$, $\alpha \geq 1$ by giving its action on each layer of the inductive limit in the following way

$$\langle f, \phi \rangle = \sum_{p,k \in \mathbb{N}_0^d} \int_{\mathbb{R}_+^d} F_{A,p,k}(x) x^{(p+k)/2} D^p \phi(x) dx + \sum_{|m| \le j, |n| \le j} \int_{\mathbb{R}_+^d} \tilde{F}_{A,n,m}(x) x^m D^n \phi(x) dx,$$

where the $L^2(\mathbb{R}^d_+)$ -functions $F_{A,p,k}$ and $\tilde{F}_{A,n,m}$ depend on the layer.

C-distribution semigroups and C-ultradistribution semigroups in locally convex spaces

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The main purpose here is investigation of the *C*-distribution semigroups and *C*-ultradistribution semigroups in the setting of sequentially complete locally convex spaces. There are a few important theoretical novelties in this field and there are some interesting examples that are given. Stationary dense operators in sequentially complete locally convex space are considered.

Strictly hyperbolic equations with low-regular coefficients with respect to time

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We discuss the strictly hyperbolic Cauchy problem

$$\begin{cases} D_t^m u = \sum_{j=0}^{m-1} \sum_{|\gamma|+j \le m} a_{m-j,\gamma}(t,x) D_x^{\gamma} D_t^j u + f(t,x), \\ D_t^{k-1} u(0,x) = g_k(x), \quad (t,x) \in [0,T] \times \mathbb{R}^n, k = 1, \dots, m, \end{cases}$$

where the coefficients of the principal part $a_{m-j,\gamma}(t,x)$, $(|\gamma|+j=m)$ satisfy a given modulus of continuity with respect to time. We derive a global (in time) existence theorem and explain how the modulus of continuity, i.e. the regularity of the coefficients in time, is linked to the regularity of the coefficients in space. Some well-known examples complete the talk.

Boehmians of L²- Growth

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Using convolution and approximate identities (delta sequences), a space β_{L^2} of generalized functions is constructed. The space of distributions D'_{L^2} of L^2 -growth is continuously embedded into β_{L^2} . The Fourier transform is a continuous linear operator from β_{L^2} into the space of locally square-integrable functions. After briefly discussing the uncertainty principle for the Fourier transform, an example of an element $W \in \beta_{L^2} \backslash D'_{L^2}$ which does not vanish on any interval is constructed.

Recent progress for wave models with scale invariant mass and dissipation

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We discuss the Cauchy problem for semi-linear wave models with scale invariant mass and dissipation and with a power non-linearity

$$u_{tt} - \Delta u + \frac{\mu_1}{1+t} u_t + \frac{\mu_2^2}{(1+t)^2} u = |u|^p, \ u(0,x) = u_0(x), \ u_t(0,x) = u_1(x)$$

where μ_1 , μ_2^2 are non-negative constants. On the one hand we explain some results for the corresponding linear model with vanishing right-hand side. On the other hand we are interested in global (in time) existence results for the semi-linear models. Here our main issue is to determine the critical exponent dividing the range of admissible exponents into those producing, in general, a blow-up behavior of solutions and those allow the proof of global existence (in time) of small data solutions. The interplay between μ_1 and μ_2 plays an important role in the study of the linear problem and consequently in the semi-linear problem, determining the different tools that results suitable in each case. Some discussion of optimality of our results and some open problems complete the talk.

Beyond Gevrey regularity

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We introduce a class of ultradifferentiable functions which describes a new type of local regularity. They contains Gevrey functions. We investigate the continuity properties of certain (ultra)differentiable operators and discuss inverse-closedness property for our classes. Finally, we define a new type of wave front sets and present our main result:

$$\operatorname{WF}_{0,\infty}(P(D)u) \subseteq \operatorname{WF}_{0,\infty}(u) \subseteq \operatorname{WF}_{0,\infty}(P(D)u) \cup \operatorname{Char}(P),$$

where u is a Schwartz distribution, P(D) is a partial differential operator with constant coefficients and WF_{0,\infty} is the wave front set described in terms of new regularity conditions.

Complex powers of infinite order pseudodifferential operators

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The study of complex powers of pseudodifferential operators started with the work of Seeley. They proved to have an essential role in the spectral analysis of elliptic operators; for example in the famous Weyl asymptotic formula. Recently, Buzano and Nicola considered complex powers of pseudodifferential operators whose symbols are merely hypoelliptic.

In this talk we discuss the problem of complex powers of the L^2 -realisations of infinite order hypoelliptic operators acting on spaces of non-quasianalytic ultradistributions. In fact, we show, under certain conditions, that these are still pseudodifferential.

The talk is based on a collaborative work with Stevan Pilipović.

Recent progress on composition in weighted modulation spaces

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In the theory of nonlinear partial differential equations we need to explain composition operators. We introduce a class of general ultradifferentiable weights for modulation spaces $\mathcal{M}_{p,q}^{w_*}$ which have at most subexponential growth. We establish analytic as well as non-analytic composition results in the spaces $\mathcal{M}_{p,q}^{w_*}$. Moreover, we study the existence of the product of two modulation spaces $M_{p,q}^s$ equipped with polynomial weights. This will give us the opportunity to treat the boundedness of composition operators acting on $M_{p,q}^s$.

Some non-standard pseudo-calculus in the treatment of dispersive equations

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Micro-local analysis, in particular, the theory of pseudo-differential or Fourier integral operators is an important tool in the treatment of qualitative properties of solutions to partial differential equations. We consider two Cauchy problems for dispersive equations. First, we are interested in the Cauchy problem

$$u_{tt} - \sum_{k,l=1}^{n} a_{k,l}(t,x)u_{x_kx_l} = f(t,x), \ u(0,x) = \varphi(x), \ u_t(0,x) = \psi(x).$$

Here the coefficients are smooth in the spatial variables, but not necessarily Lipschitz continuous in the time variable. Our goal is the construction of a parametrix. The second model is the Cauchy problem for the degenerate Schrödinger equation

$$iu_t + t^l \Delta_x u + \sum_{j=1}^n t^k b_j(t, x) u_{x_j} = 0, \ u(0, x) = \varphi(x).$$

Here our goal is to study well-posedness in Gevrey spaces, in particular, the influence of Levi conditions on the Gevrey exponent. For both problems we develop a non-standard pseudo-calculus basing on a division of the extended phase space into zones.

Small test function and large distribution spaces, invariant under Fourier transformation

JOACHIM TOFT

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We consider a family of function spaces, defined by estimates of $H^N f$, which include all Fourier invariant standard Gelfand-Shilov (FIGS) spaces. Here H is the harmonic oscillator. Such approach was done by Pilipović in the 80th, and therefore the function spaces are called Pilipović spaces. The smallest spaces are significantly smaller than the non-trivial FIGS-spaces, and family of corresponding distribution spaces contain spaces larger any FIGS distribution space. We consider:

- Characterizations in terms of Hermite expansions and Bargmann transform.
- A "Paley-Wiener related" property", linking Grchenig's S_C space to the family
- Estimates like

$$\sigma_j(T) \lesssim e^{-c \cdot j^{\frac{1}{2ds}}}$$

for singular values of operators with kernels in the family.

Parts of the talk is based on collaborations with Y. Chen and M. Signahl.

Theory and applications of time-frequency analysis

VILLE TURUNEN

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When and how often something happens in a signal? By properly quantizing these questions, we obtain the Born–Jordan time-frequency transform, defining a sharp phase-space energy density. We study properties of different time-frequency transforms, and also present computed examples from acoustic signal processing, quantum mechanics and medical sciences.

On general Stieltjes moment problems

JASSON VINDAS

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The problem of moments, as its generalizations, is an important mathematical problem which has attracted much attention for more than a century. It was first raised and solved by Stieltjes for positive measures. Boas and Pólya, independently, showed later that given an arbitrary sequence $\{a_n\}_{n=0}^{\infty}$ there is always a function of bounded variation F such that

$$a_n = \int_0^\infty x^n \mathrm{d}F(x) \,, \quad n \in \mathbb{N} \,. \tag{1}$$

A major improvement to this result was achieved by Durán, who was able to show the existence of regular solutions to (1). He proved that in fact every Stieltjes moment problem

$$a_n = \int_0^\infty x^n \phi(x) dx, \quad n \in \mathbb{N},$$
 (2)

admits a solution $\phi \in \mathcal{S}(0, \infty)$, that is, a solution in the Schwartz class of rapidly decreasing smooth functions with supp $\phi \subseteq [0, \infty)$.

In this talk we discuss an approach to abstract moment problems that leads to "if and only if" criteria for solvability. In particular, we shall replace the sequence of monomials in (2) by a rather general sequence of distributions $\{f_n\}_{n=0}^{\infty}$ with supp $f_n \subseteq [0,\infty)$ and present a complete characterization of those $\{f_n\}_{n=0}^{\infty}$ such that every generalized Stieltjes moment problem

$$a_n = \langle f_n, \phi \rangle$$
, $n \in \mathbb{N}$,

has a solution $\phi \in \mathcal{S}(0,\infty)$. The talk is based on collaborative works with R. Estrada, and A. Debrouwere.

Eigenexpansions of ultradifferentiable functions and ultradistributions in \mathbb{R}^n

ĐORĐE VUČKOVIĆ

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In this talk we will show a characterization of $\mathcal{S}^{\{M_p\}}_{\{M_p\}}(\mathbb{R}^n)$ and $\mathcal{S}^{(M_p)}_{(M_p)}(\mathbb{R}^n)$, the general Gelfand-Shilov spaces of ultradifferentiable functions of Roumieu and Beurling type, in terms of decay estimates for the Fourier coefficients of their elements with respect to eigenfunction expansions associated to normal globally elliptic differential operators of Shubin type. Moreover, we will show that the eigenfunctions of such operators are absolute Schauder bases for these spaces of ultradifferentiable functions. Our characterization extends earlier results by Gramchev et al. (Proc. Amer. Math. Soc. 139 (2011), 4361–4368.) for Gevrey weight sequences. It also generalizes to \mathbb{R}^n recent results by Dasgupta and Ruzhansky, which were obtained in the setting of compact manifolds.

This talk is based on collaborative work with Jasson Vindas (J. Pseudo-Diff. Oper. Appl. (2016), doi: 10.1007/s11868-016-0157-9).

WORKSHOP 2

Fixed point theory and its applications

Workshop Description:

The purpose of the special session on "Fixed point theory and its applications" is to bring together leading experts and researchers in fixed point theory and to assess new developments, ideas and methods in this important and dynamic field. A special emphasis will be put on applications in related areas, as well as other sciences, such as the natural sciences, medicine, economics and engineering.

Organizer and workshop leader:

• Erdal Karapinar, Atilim University, Ankara, Turkey (erdalkarapinar@yahoo.com) / primary contact.

Duration: One day.

Target Audience: Basic analysis and topology is enough to follow

lectures.

Learning Objectives:

1. Recent improvement in fixed point theory will be discussed.

2. Application in different areas will be discussed.

Teaching Methods: Lectures and discussing groups.

Participants:

- 1. Erdal Karapinar, Atilim University, Ankara, Turkey
- 2. Inci Erhan, Atilim University, Ankara, Turkey
- 3. Selma Gulyaz, Cumhuriyet University, Sivas, Turkey
- 4. İshak Altun, Department of Mathematics, College of Science, King Saud University, Saudi Arabia and Department of Mathematics, Faculty of Science and Arts, Kirikkale University, Turkey
- 5. Hacer Dağ, Departmento de Matematica Aplicada, Universitat Politecnica de Valencia, Spain
- 6. Hakan Şimşek, Department of Mathematics, Faculty of Science and Arts, Kirikkale University, Turkey
- 7. Gonca Durmaz, Department of Mathematics, Faculty of Science, Çankırı Karatekin University, Turkey
- 8. Murat Olgun, Department of Mathematics, Faculty of Science, Ankara University, Ankara, Turkey
- 9. Mahpeyker Öztürk, Department of Mathematics, Faculty of Science, Sakarya University, Turkey
- 10. Umit Aksoy, Atilim University, Ankara, Turkey
- 11. Mahpeyker Ozturk, Sakarya University, Turkey
- 12. Mehdi Asadi, Azadi University, Iran
- 13. Antonio Roldan, Jaén University, Spain
- 14. Hakan Şimşek, Kirikkale University, Turkey
- 15. Murat Olgun, Ankara University, Turkey
- 16. Ali Uçum, Kirikkale University, Turkey
- 17. Rezan Sevinik Adıgüzel, Atilim University, Turkey

Applications of fixed point theory results to ordinary and partial differential equations

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In this talk, a class of weak ψ -contractions satisfying C-condition on partially ordered metric spaces and α -admissible contractions defined on b-metric spaces are presented with existence and uniqueness of their fixed points. The results are employed in the discussion of solutions of periodic boundary value problems for ordinary differential equations and boundary value problems for partial differential equations.

Some fixed point results on quasi metric spaces

ISHAK ALTUN

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Fixed point theory is one of basic subjects in the mathematical analysis and topology. Existence and uniqueness of solutions of differential and integral equations can be find by fixed point theory. Therefore, an operator is determined for differential and integral equation, and one to one correspondence is constructed between the existence (and the uniqueness) of fixed point of this operator and existence (and uniqueness) solutions of differential or integral equations. This theory, which is useful in application, contains the research about whether the fixed point of an operator exist, whether the fixed point is unique, if it is unique, then how to find it.

Fundamentally, fixed point theory divides into three major subjects which are topological, discrete and metric. However, this theory has been improving on the metric because of useful of the applications.

The purpose of this talk is to present some fixed point theorems for single valued mappings considering a new type contraction on some kind of complete quasi metric spaces, which has a comprehensive structure space and has a more application on computer science and semantics.

- [1] I. Altun, G. Mınak and M. Olgun, Classification of completeness of quasi metric space and some new fixed point results, Submitted.
- [2] E. Karapınar and B. Samet, Generalized α - ψ -contractive type mappings and related fixed point theorems with applications, Abstr. Appl. Anal. 2012, Article ID 793486, 17 pp.
- [3] S. Romaguera, Left K-completeness in quasi-metric spaces, Math. Nachr., 157 (1992), 15-23.
- [4] D. Wardowski, Fixed points of a new type of contractive mappings in complete metric spaces, Fixed Point Theory Appl., 2012, 2012:94, 6 pp.

A fixed point theorem for multivalued mappings on quasi metric space

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The purpose of this talk is to provide some new fixed point results for multivalued mapping on some kind of complete quasi metric spaces by taking into account recent contractive technique, which is called Fcontraction.

- [1] I. Altun, M. Olgun and G. Mınak, Classification of completeness of quasi metric space and some new fixed point results, Submitted.
- [2] S. Cobzaş, Completeness in quasi-metric spaces and Ekeland variational principle, Topol. Appl., 158 (2011), 1073-1084.
- [3] H. Dag, G. Minak and I. Altun, *Some fixed point results for mutlivalued F-contractions on quasi metric spaces*, to appear in RACSAM (2016).
- [4] Y. Feng and S. Liu, Fixed point theorems for multi-valued contractive mappings and multi-valued Caristi type mappings, J. Math. Anal. Appl., 317 (2006), 103-112.
- [5] D. Wardowski, *Fixed points of a new type of contractive mappings in complete metric spaces*, Fixed Point Theory Appl., 2012, 2012:94, 6 pp.

Some relations between recent fixed point results on metric space

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The aim of this talk is to take into account the effect of simulation functions to existence of fixed points of single valued mappings and introduce some nontrivial examples of the function.

- [1] H. Argoubi, B. Samet, C. Vetro, *Nonlinear contractions involving simulation functions in a metric space with a partial order*, J. Nonlinear Sci. Appl., 8 (2015), 1082-1094.
- [2] G. Durmaz, I. Altun, Fixed point results for α -admissible mappings with modified simulation functions, Submitted.
- [3] G. Durmaz, G. Mınak and I. Altun, Fixed point results for α - ψ -contractive mappings including almost contractions and applications, Abstr. Appl. Anal., 2014, Article ID 869123, 10 pp.
- [4] F. Khojasteh, S. Shukla and S. Radenović, *A new approach to the study of fixed point theory for simulation functions*, Filomat 29:6 (2015), 1189-1194.
- [5] M. Olgun, T. Alyıldız and Ö. Biçer, *A new aspect to Picard operators with simulation functions*, Turkish Journal of Mathematics, DOI: 10.3906/mat-1505-26.
- [6] A. Roldan, E. Karapınar, C. J. Roldan and M. Moreno, *Coincidence point theorems on metric spaces via simulation functions*, J. Comput. Appl. Math., 275 (2015), 345-355.

Quasi-dislocated spaces

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In this talk a class of generalized α -admissible contraction mappings on quasi-b-dislocated spaces are defined. The study is inspired by recent fixed point and Ulam-Hyers stability results on b-metric spaces [1, 2] and quasi-b-dislocated spaces [3]. Existence and uniqueness of fixed points for this class of mappings is discussed. Various consequences of the main results are given. Ulam-Hyers stability and generalized Ulam-Hyers stability are defined on quasi-b-dislocated spaces and related Ulam-Hyers stability results for fixed point problems are stated.

The results presented in the talk extend and generalize some existing results in the literature.

- [1] M. Bota, E. Karapınar and O. Mlesnite, *Ulam-Hyers stability results for fixed point problems via* α-ψ-contractive mapping in (b)-metric space, Hindawi Publishing Corporation, Abstract and Applied Analysis, vol. 2013, Article ID 825293, 6 p., 2013.
- [2] V. L. Lazar, *Ulam-Hyers stability for partial differential inclusions*, Electronic Journal of Qualitative Theory of Differential Equations, vol. 21 (2012) 1-19.
- [3] S. Gülyaz, Fixed points of $\alpha \psi$ contraction mappings on quasi-b-metric-like spaces, Yokohama Publischer, 2016, in press.

Advances on fixed point theory

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One of the most dynamic research subjects in nonlinear analysis is the fixed point theory. In particular, metric fixed point theory has been investigated by a number of researchers since the theory has a wide application potential to the several applied science besides its own interest. According to needs of the application problems, the researchers in this fields have proposed several fixed point results and also extend the standard frame, complete metric space, to the more general abstract spaces, like, quasi-metric space, b-metric space, dislocated metric space and so on.

The aim of this talk is to put a general frame that covers almost all recent results in the context of metric fixed point theory.

Modular spaces and some fixed point theorem

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The aim of this paper is to present fixed point results of mappings satisfying various generalized contractive conditions in the framework of modular spaces. Also by using the concept given by Jachymski [Proc. Amer. Math. Soc., 136 (2008), 1359–1373], we obtained some results in modular spaces endowed with a graph. Some examples are presented to support the results proved herein.

Some properties of the q-quadratic lattices

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The central idea of this talk is to introduce some differences between the properties for q-linear and q-quadratic lattices. Such lattices are defined for the second order linear difference equation of hypergeometric type. Polynomial solutions of such difference equations are well known. Their theory are important in mathematcal physics.

On classification of bi-null curves in \mathbb{R}^6_3

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In [1], the author introduced a new type of null curves called "bi-null curves" in \mathbb{R}_2^n . In this paper, we give the notion of bi-null curves in \mathbb{R}_3^6 . In addition, we define osculating, normal and rectifying bi-null curve and also we give the necessary and sufficient conditions for bi-null curves in \mathbb{R}_3^6 to be osculating, normal or rectifying curves in terms of their curvature functions.

- [1] M. Sakaki, *Bi-null Cartan curves in semi-Euclidean spaces of index 2*, Beitr. Algebra Geom., 53 (2012), no. 2, 421–436.
- [2] K. İlarslan and E. Nesovic, *Some characterizations of null, pseudo null and partially null rectifying curves in Minkowski space-time,* Taiwanese J. Math. 12 (2008), no. 5, 1035–1044
- [3] K. İlarslan, E. Nesovic, *The first kind and second kind osculating curves in Minkowski space-time*, Compt. rend. Acad. Bulg. Sci., 62(6) (2009), 677-686.
- [4] K. İlarslan, E. Nesovic, Some relations between normal and rectifying curves in Minkowski space-time, Int. Electron. J. Geom. 7 (2014), no. 1, 26-35.

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Vera Dimovska,

Combinatorial game theory: An analysis of a fair game

Paolina Gadjulova,

Simulation of an ecosystem with three species on a toroidal lattice

Vanessa Getseva,

Haplotype phasing of structural variations in genomes of individuals

Filip Selamovski,

Analysis of terrorism through game theory

Combinatorial game theory: An analysis of a fair game

VERA DIMOVSKA

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The game "The princess and the roses" is a combinatorial game and is still an open problem. In the following presentation I have made an analysis of the game. For up to 5 bushes I have proven mathematically the strong bonds with parity, also using MathLab I have made a graphic representation of some 6 bush situations. I tried to find a connection between the number of roses and their divisibility with 3 and 4. Furthermore I found all of the winning positions by fixing the sixth bush to have 2 roses by using my program and made a modified version of the game. Potential future work would mean finding a winning strategy for 6 or more bushes or a potential wining equation or strategy for any situation.

Analysis of terrorism through game theory

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The theme of this project is from the area of applied mathematics. The aim is to examine how game theory can be applied, so that we can detect the best strategies, through which the target of the attack can handle the terrorism, with least possible damage. In this project we will give a new view to this field of interest, through real scenarios and situations and we will give unanswered questions, that are connected with our project.

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